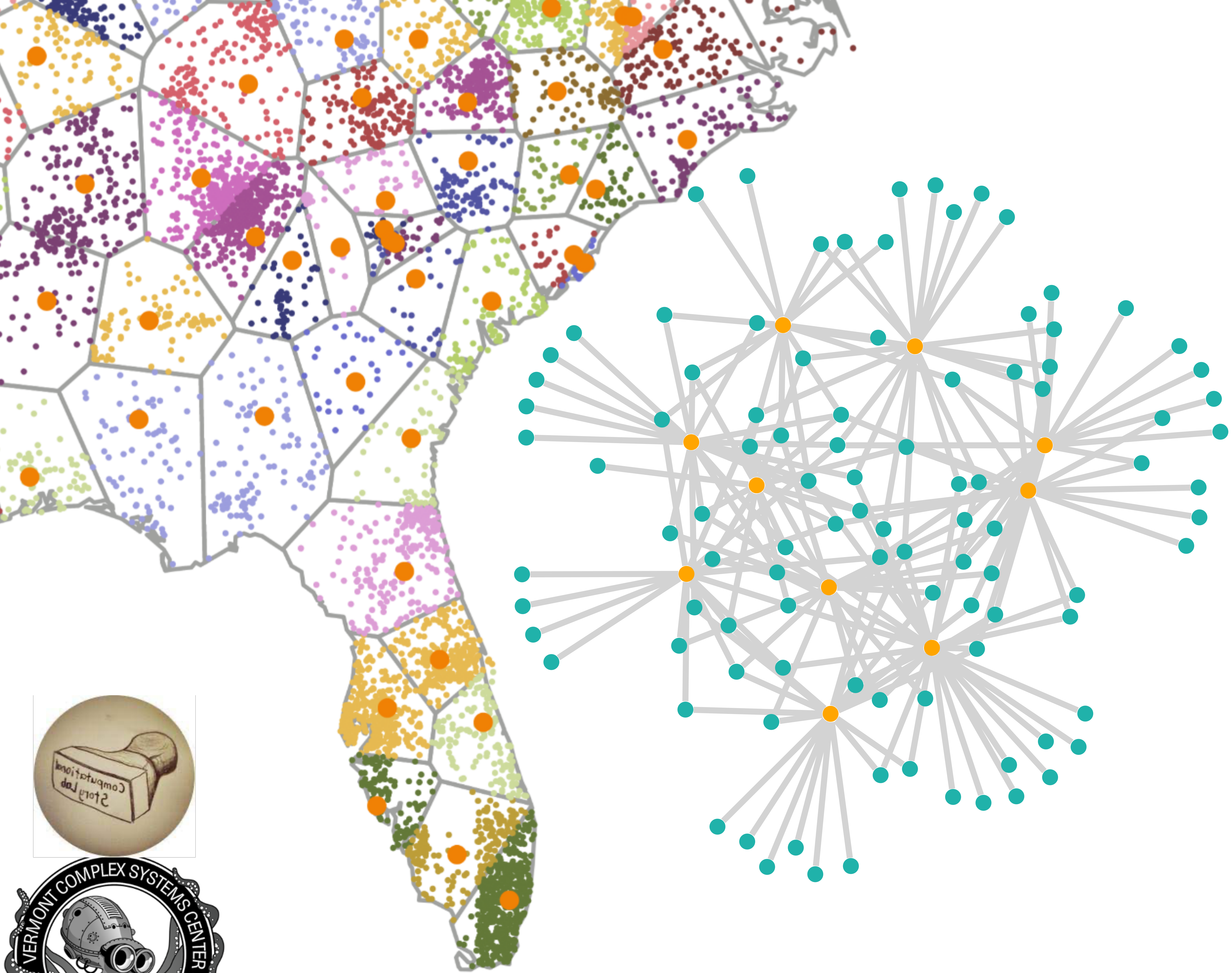
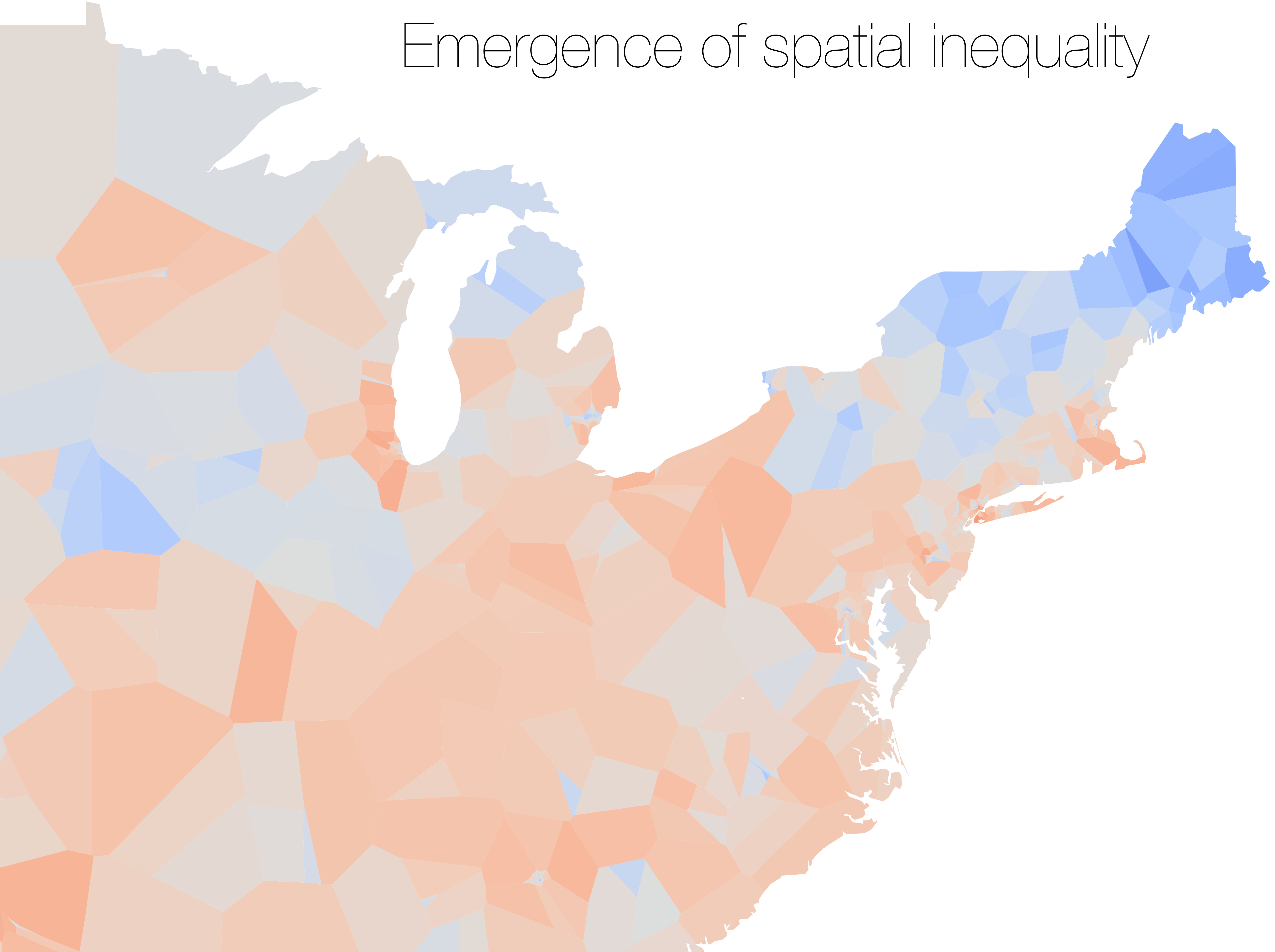


Modeling the Origins of Inequality

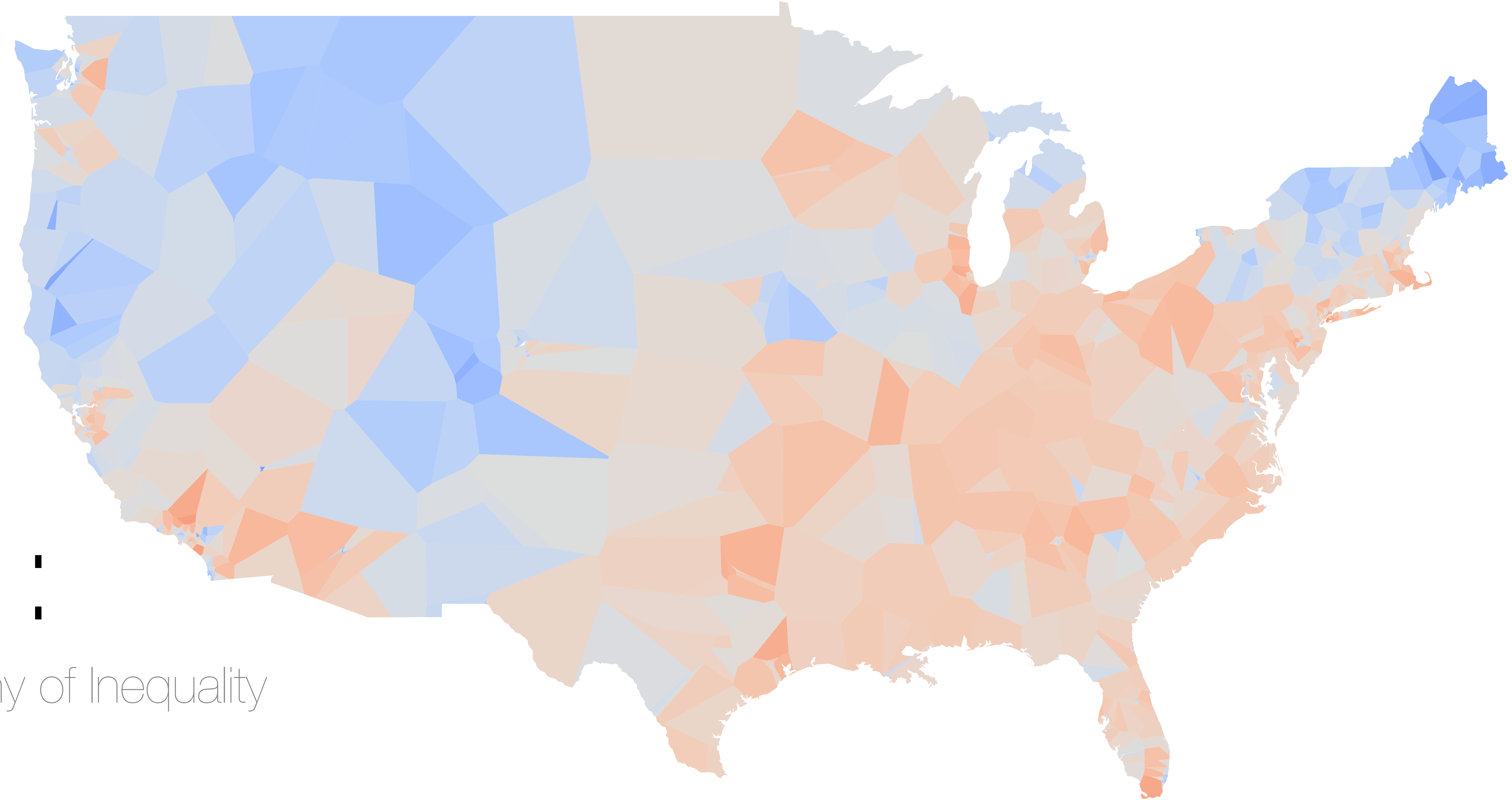
Will Thompson
10/31/23

How does Inequality Emerge in Spatial and Social Systems?

Emergence of spatial inequality



Emergence of inequality through political polarization

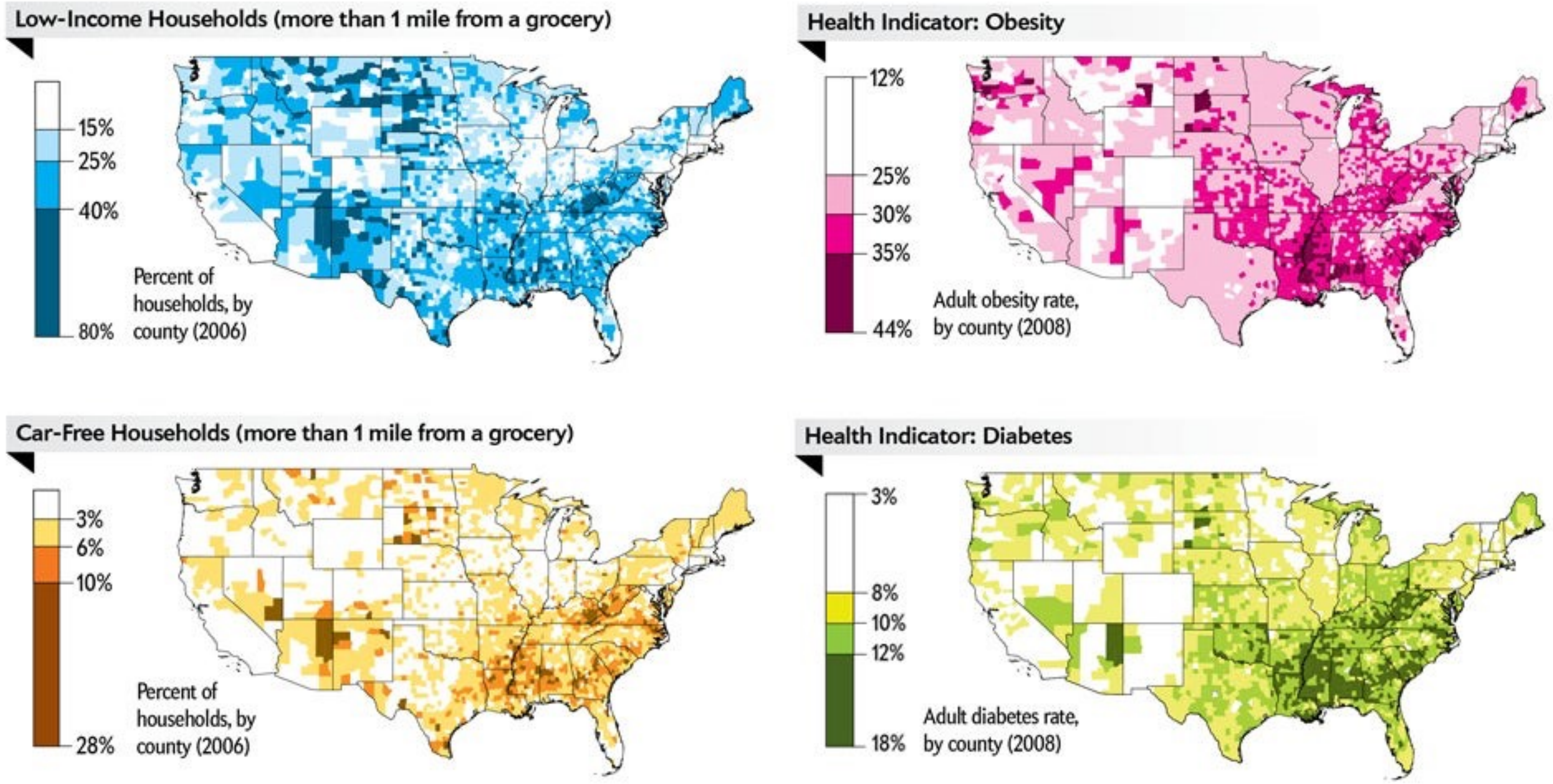


Part 1:

The Geography of Inequality

Spatial Inequality

- **Spatial Inequality**: social Inequality the arises from the unequal distribution of resources
- Social Deserts: spatial regions with limited access to socially important goods and services
  - Food Deserts: Low access to nutritious food affects 39.5 million Americans[1]. Strong correlation between access to nutritious food and health outcomes
  - Social deserts for books [2], transit [3] and other socially important goods

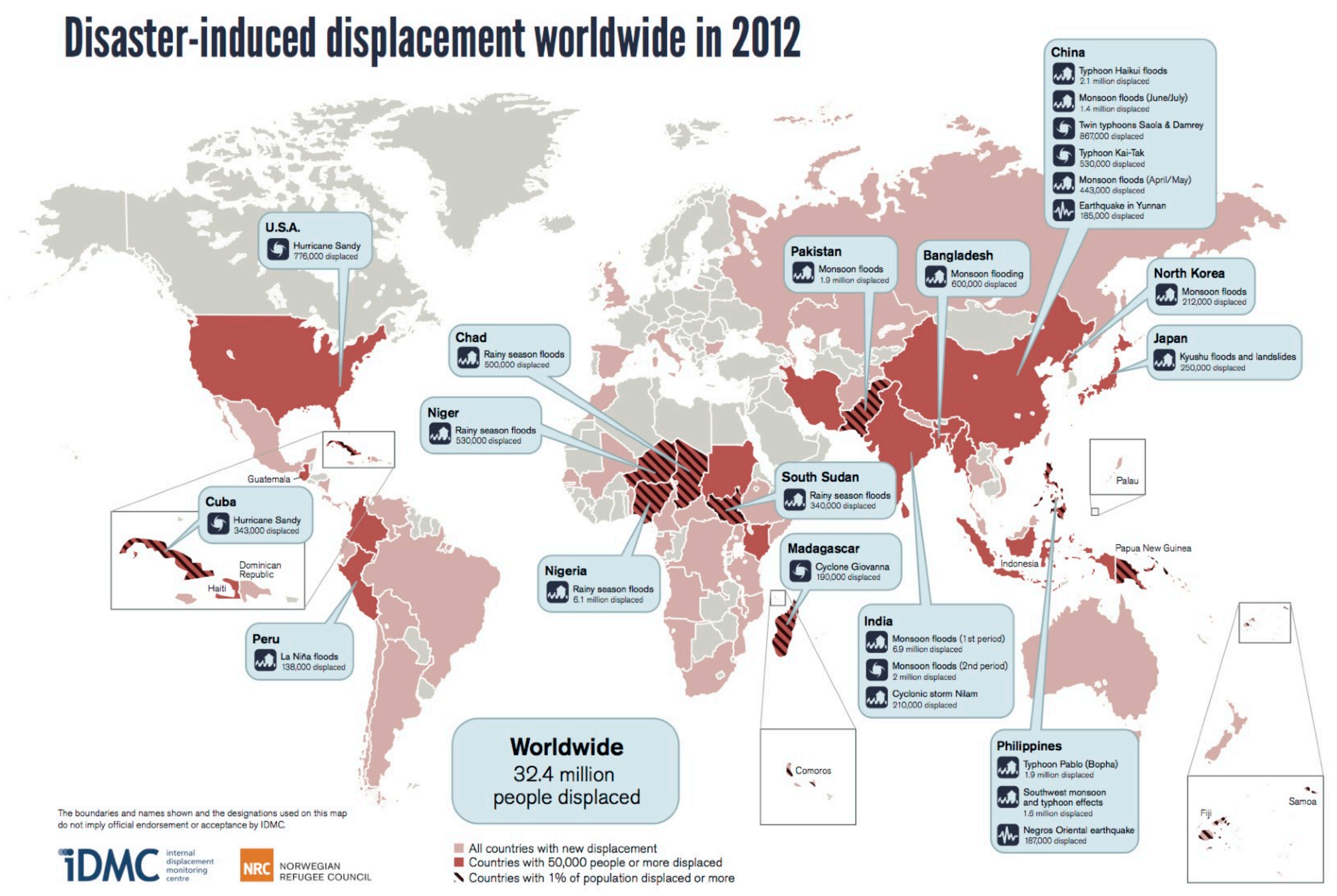


Source: Food Environment Atlas, U.S. Department of Agriculture, Economic Research Service

[1]M. V. Ploeg et al., "Access to Affordable and Nutritious Food: Measuring and Understanding Food Deserts and Their Consequences,"
[2] S. B. Neuman and N. Moland, "Book Deserts: The Consequences of Income Segregation on Children's Access to Print," Urban Education, vol. 54, no. 1, pp. 126–147, Jan. 2019
[3]J. Jiao and M. Dillivan, "Transit Deserts: The Gap between Demand and Supply," JPT, vol. 16, no. 3, pp. 23–39, Sep. 2013

Changing systems develop inequality

- Spatial resources are affected by changes in demand and changes in supply.
- **Changes in demand**
  - Century long rural decline reversed by COVID
  - Climate/disaster induced displacement[1] will displace 250 million people by 2050
- **Changes in supply**:
  - Natural disasters.
    - Hurricane Katrina and Sandy[2]
  - Policy induced changes in supply
    - Dobbs and Abortion Access

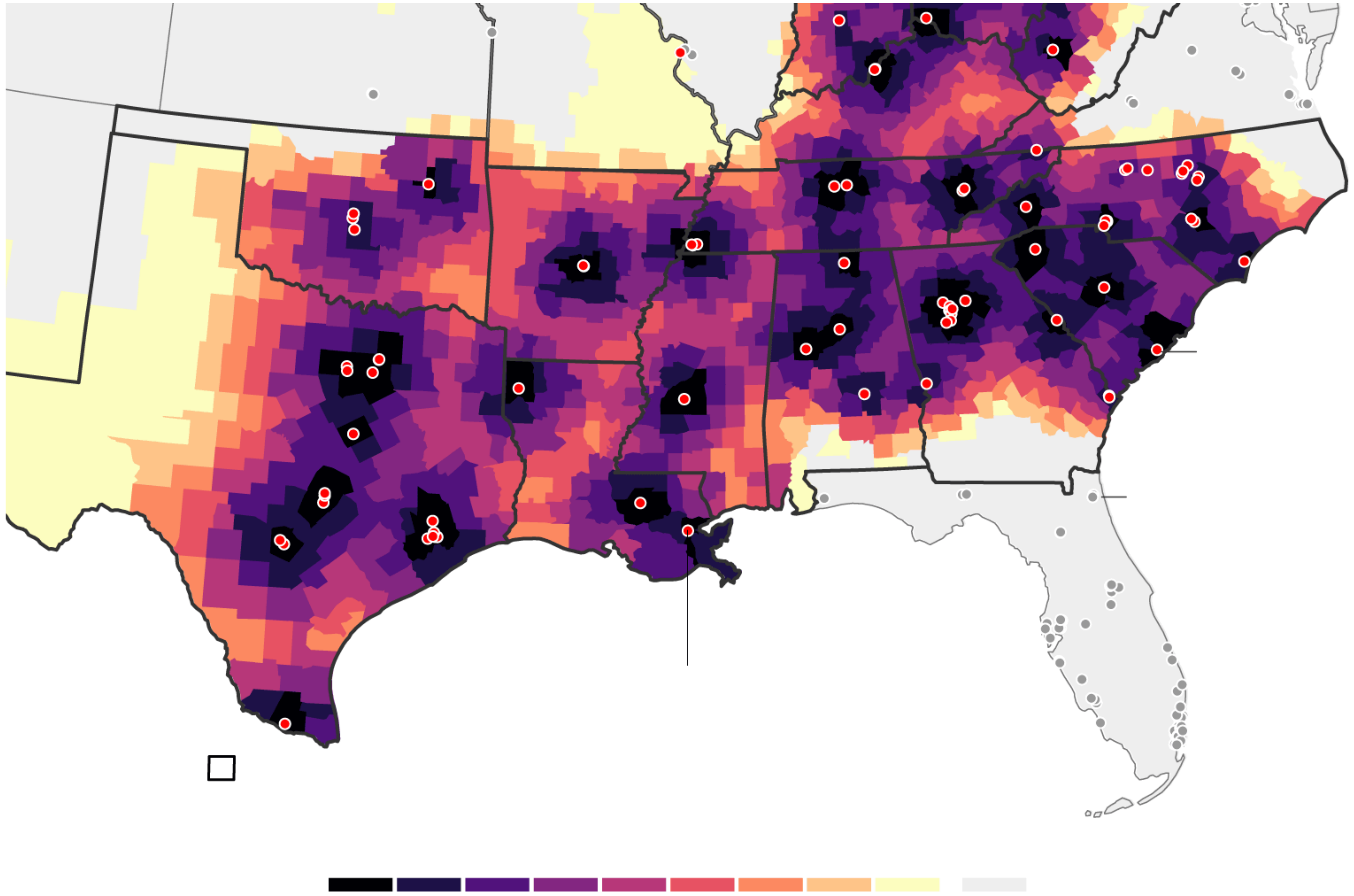


[1] Katharine J. Mach and A. R. Siders, "Reframing Strategic, Managed Retreat for Transformative Climate Adaptation", Science 372, no. 6548 (2021): 1294-1299
[2] Mary W. Chaffee, Neill S. Oster, and Associate Editors. "The Role of Hospitals in Disaster". en. In: Disaster Medicine (2006). Publisher: Elsevier

Policy Induced Changes: Abortion Clinics

- While in 2008 the median distance traveled to an abortion clinic was only 15 miles, some women had to travel much farther. 17% of woman needed to travel at least 50 miles to the nearest clinics[3]
- In Texas and Louisiana:
  - Pre Dobbs median commute: 27 mins
  - Post Dobbs median commute: 6 hours

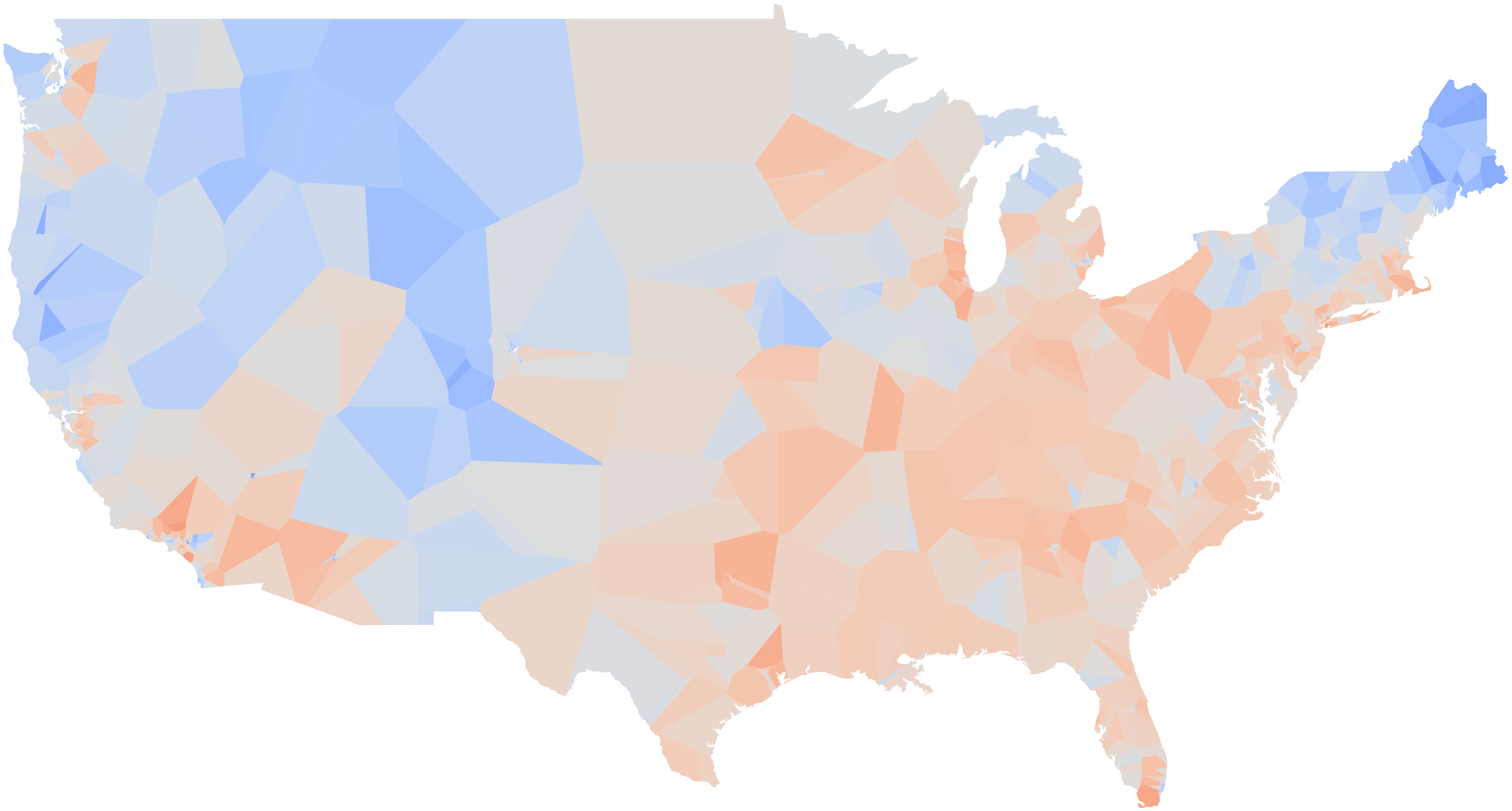
Predicted Decline In Legal Abortions

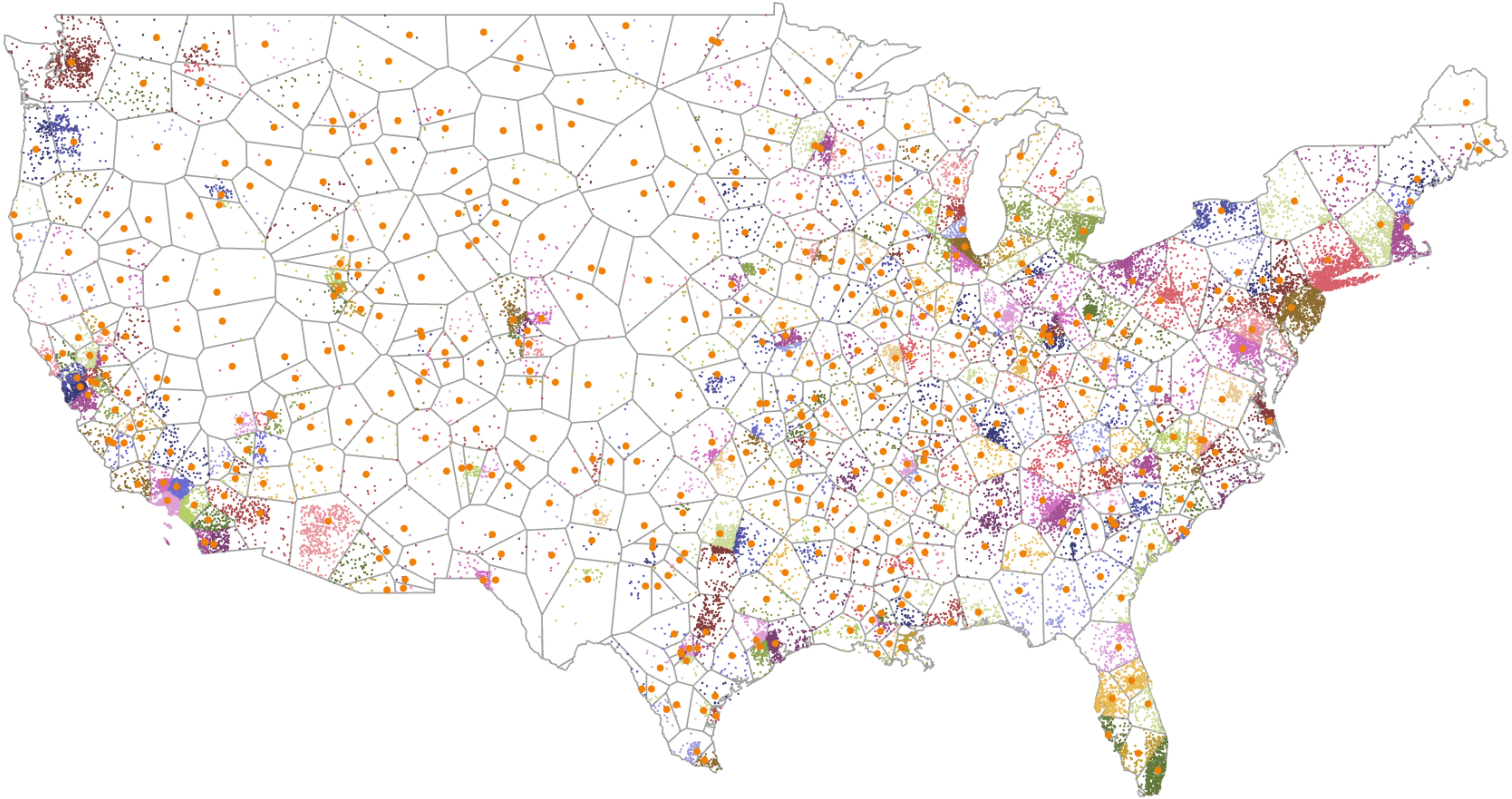


New York Times: Where Abortion Access Would Decline if Roe V. Wade Were Overturned(May 2021)

[3] J. M. Lindo, C. K. Myers, A. Schlosser, and S. Cunningham, "How Far Is Too Far? New Evidence on Abortion Clinic Closures, Access, and Abortions," J. Human Resources, vol. 55, no. 4, pp. 1137–1160, Oct. 2020

Research Question: Can we map the geography of inequality?
What can optimal facility allocation problems tell us about the misallocation of facilities





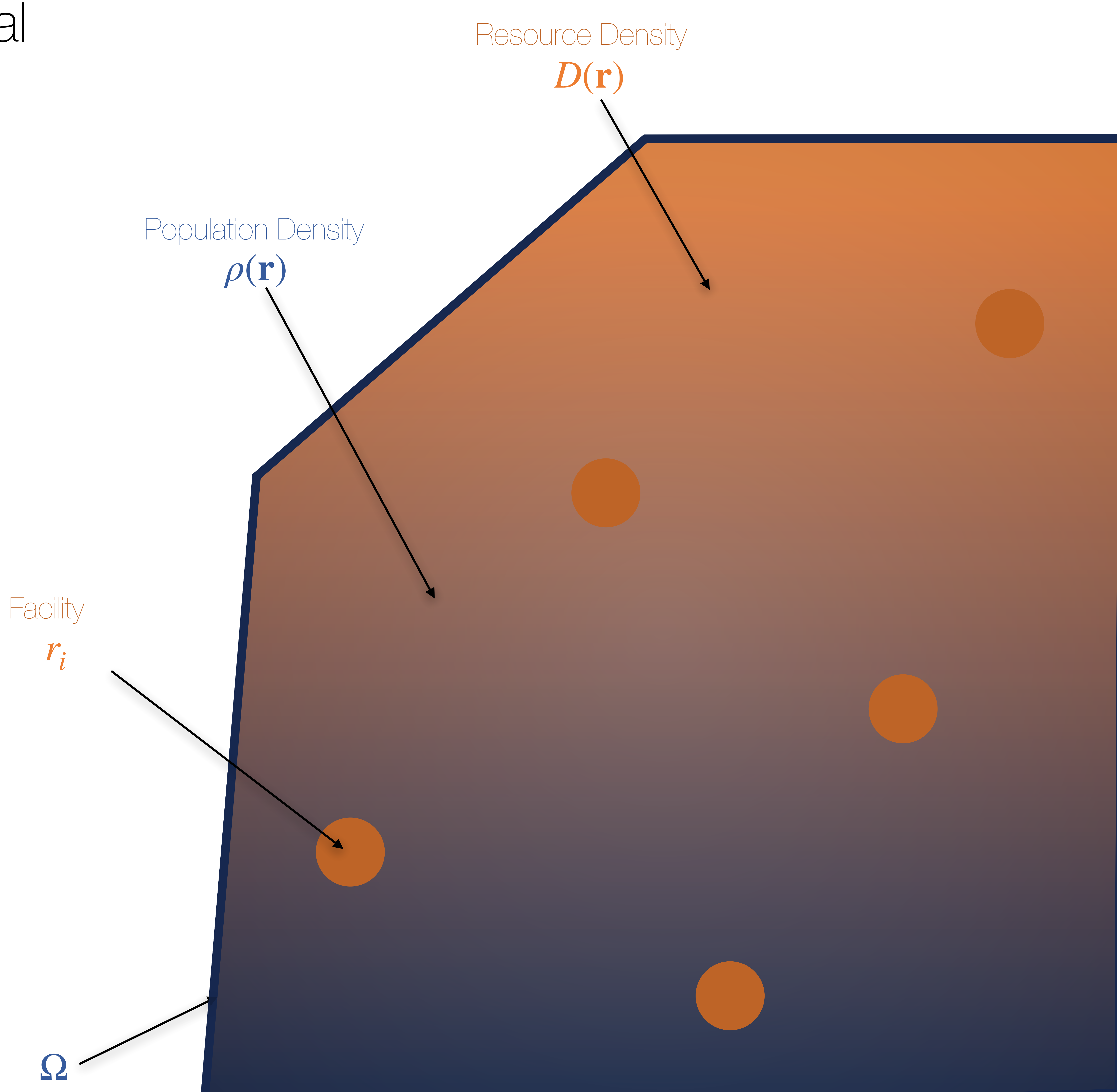
Optimal Facility Allocation

Formalism: How do we allocate spatial resources optimally?

- Given a population density $\rho(\mathbf{r})$ and a bounded region $\boldsymbol{\Omega}$
- A resource can be allocated over $\boldsymbol{\Omega}$, specified by $D(\mathbf{r})$
- If resources as discrete set of facilities. Imagine p facilities $\{r_1 \ldots r_p\}$:

$$D(\mathbf{r}) = \sum_{i=1}^{p} \delta(\mathbf{r} - \mathbf{r_i})$$

- The Problem: Find a resource density $D(\mathbf{r})$ which extremizes some objective functional $F[D(\mathbf{r})]$



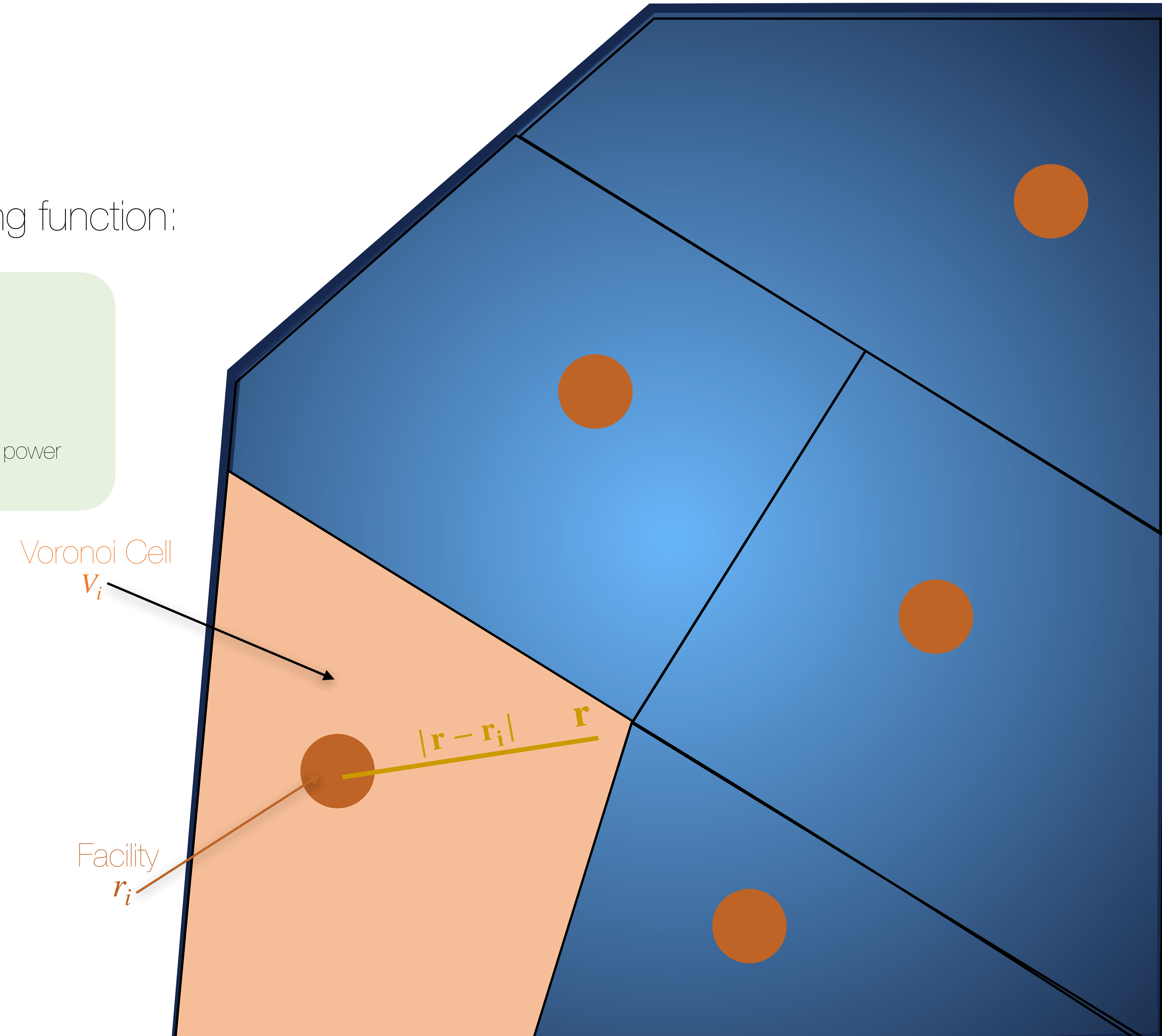
- Assume each person goes to nearest facility.
  Area of coverage is the Voronoi cell V_i.

P Median Problem

We want to find the set of p facilities that extremes the following function:

$$F(\mathbf{r}_1, \mathbf{r}_2 \ldots \mathbf{r}_p) = \sum_{i=1}^{p} \int_{V_i} d\mathbf{r} \rho(\mathbf{r}) \, |\, \mathbf{r} - \mathbf{r_i} \,|^{\beta}$$

Objective function is **population weighted average distance to nearest facility** to some power



$$F(\mathbf{r}_1, \mathbf{r}_2 \ldots \mathbf{r}_p) = \sum_{i=1}^{p} \int_{V_i} d\mathbf{r} \rho(\mathbf{r}) |\mathbf{r} - \mathbf{r_i}|^{\beta}$$

Objective function is **population weighted average distance to nearest facility** to some power

- How do we solve this? - Approximations!
- When the number of facilities p is sufficiently large, the population of each cell is constant:

$$|\mathbf{r} - \mathbf{r_i}| \approx \langle |\mathbf{r} - \mathbf{r_i}| \rangle = g[s(\mathbf{r})^{1/2}]$$

g is a shape factor of $\mathcal{O}(1)$

Returns area of cell that $\mathbf{r}$ is in

$$s(\mathbf{r}) = \int_{V_i} d\mathbf{r}$$

$$D(\mathbf{r}) = \frac{1}{s(\mathbf{r})}$$

$$F = \int_{\Omega} d\mathbf{r} g^{\beta} \rho(\mathbf{r}) [s(\mathbf{r})]^{\beta/2} + \lambda \left(\int_{\Omega} \frac{d\mathbf{r}}{s(\mathbf{r})} - p \right)$$

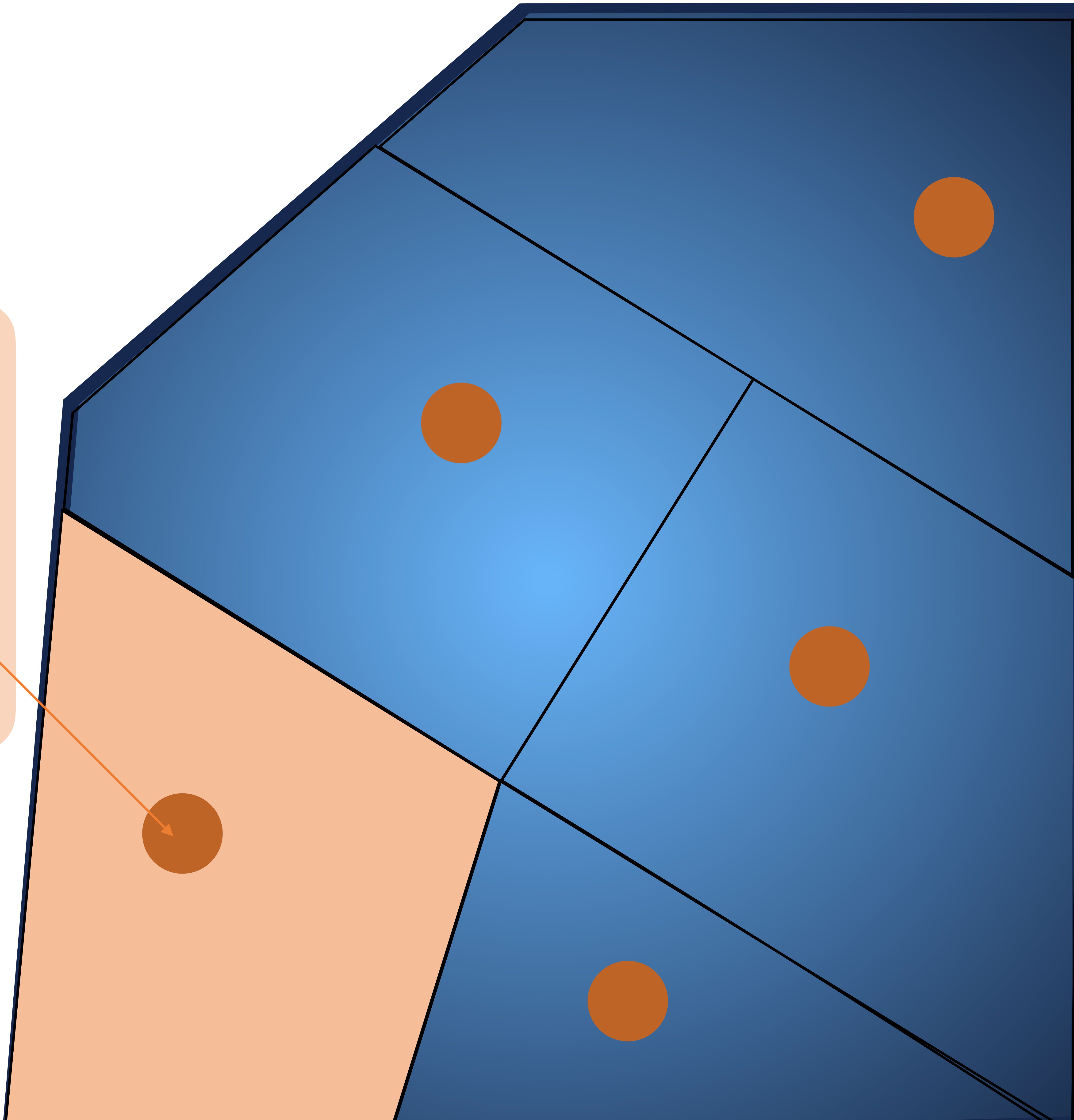
Average distance to the nearest facility to some power β

Constraint: p facilities

When is our objective function is extremized$(\delta F = 0)$

when the above **power-law relation** exists between the **facility density $D(\mathbf{r})$** and the **population density $\rho(\mathbf{r})$**

$$D(\mathbf{r}) = \frac{1}{s(\mathbf{r})} = \propto \rho(\mathbf{r})^{\frac{2}{\beta+2}}$$



Objective function is **population weighted average distance to nearest facility** to some power

$$F(\mathbf{r}_1, \mathbf{r}_2 \ldots \mathbf{r}_p) = \sum_{i=1}^{p} \int_{V_i} d\mathbf{r}\rho(\mathbf{r}) |\mathbf{r} - \mathbf{r_i}|^{\beta}$$

$$D(\mathbf{r}) = \frac{1}{s(\mathbf{r})} = \propto \rho(\mathbf{r})^{\frac{2}{\beta+2}}$$

When our objective function is extremized when the above **power-law relation** exists between the **facility density** $D(\mathbf{r})$ and the **population density** $\rho(\mathbf{r})$

What does the objective function represent?

F is **minimized** ($\delta^2 F > 0$) for $\beta > 0$

If $\beta = 1$

$$F(\mathbf{r}_1, \mathbf{r}_2 \ldots \mathbf{r}_p) = \sum_{i=1}^{p} \int_{V_i} d\mathbf{r}\rho(\mathbf{r}) \langle |\mathbf{r} - \mathbf{r_i}| \rangle$$

Average distance to nearest facility

When $\beta = 1$ we **minimize** the **average distance to the nearest facility**

$$D(\mathbf{r}) \propto \rho(\mathbf{r})^{\frac{2}{1+2}} = \rho(\mathbf{r})^{\frac{2}{3}}$$

An optimal facility density $D(\mathbf{r})$ should scale as the **population** $\rho(\mathbf{r})$ **to the 2/3**

$\beta = 1$ **minimizes social opportunity cost** we should see this for **public facilities**

F is **maximized** ($\delta^2 F < 0$) for $\beta \in (-2,0)$

If $\beta = 0$

$$F(\mathbf{r}_1, \mathbf{r}_2 \ldots \mathbf{r}_p) = \sum_{i=1}^{p} \int_{V_i} d\mathbf{r}\rho(\mathbf{r})$$

Total cell population

When $\beta = 0$ we maximize the total cell population

$$D(\mathbf{r}) \propto \rho(\mathbf{r})^{\frac{2}{0+2}} = \rho(\mathbf{r})^1$$

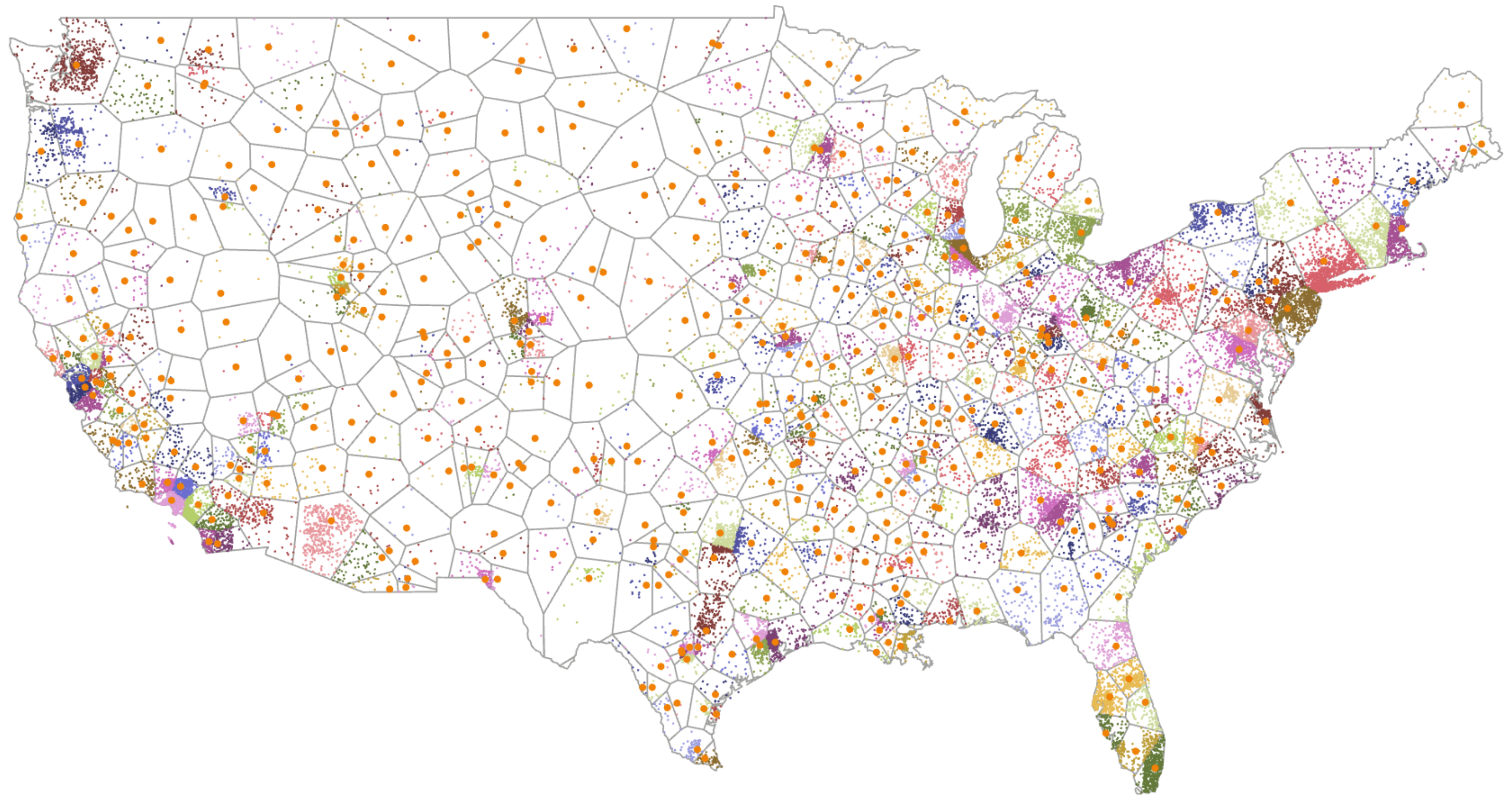
An optimal facility density $D(\mathbf{r})$ should scale as the **population** $\rho(\mathbf{r})$

$\beta = 0$ **maximizes profit** we should see this for **commercial facilities**

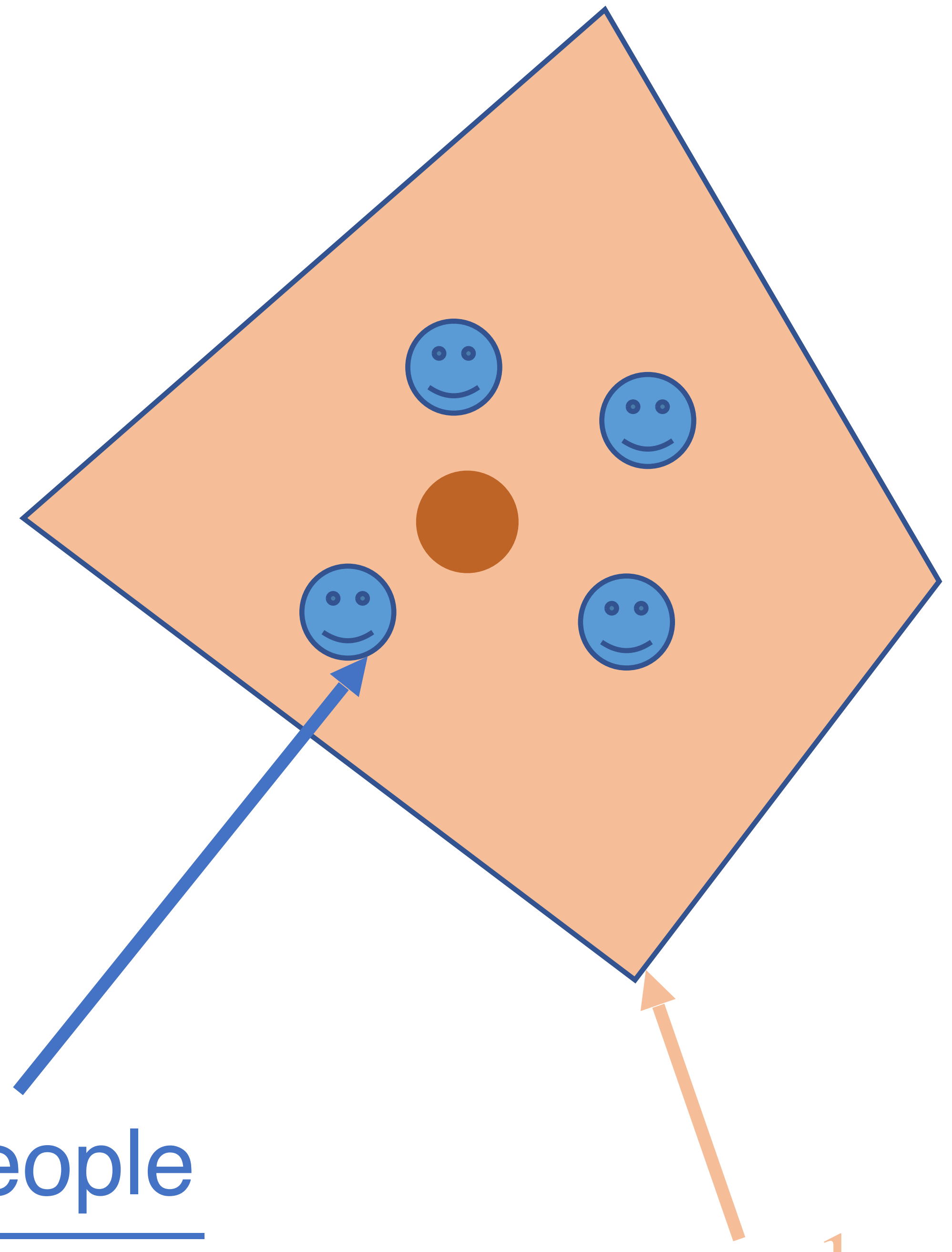
[4] J. I. Park and B. J. Kim, "Generalized p-median problem for the optimal distribution of facilities," J. Korean Phys. Soc., vol. 80, no. 4, pp. 352–358, Feb. 2022, doi: 10.1007/s40042-021-00361-2.
[5] J. Um, S.-W. Son, S.-I. Lee, H. Jeong, and B. J. Kim, "Scaling laws between population and facility densities," Proceedings of the National Academy of Sciences, vol. 106, no. 34, pp. 14236–14240, Aug. 2009, doi: 10.1073/pnas.0901898106.

Quantifying Facility Misallocation

1: Create Voronoi Cells based on facility placements



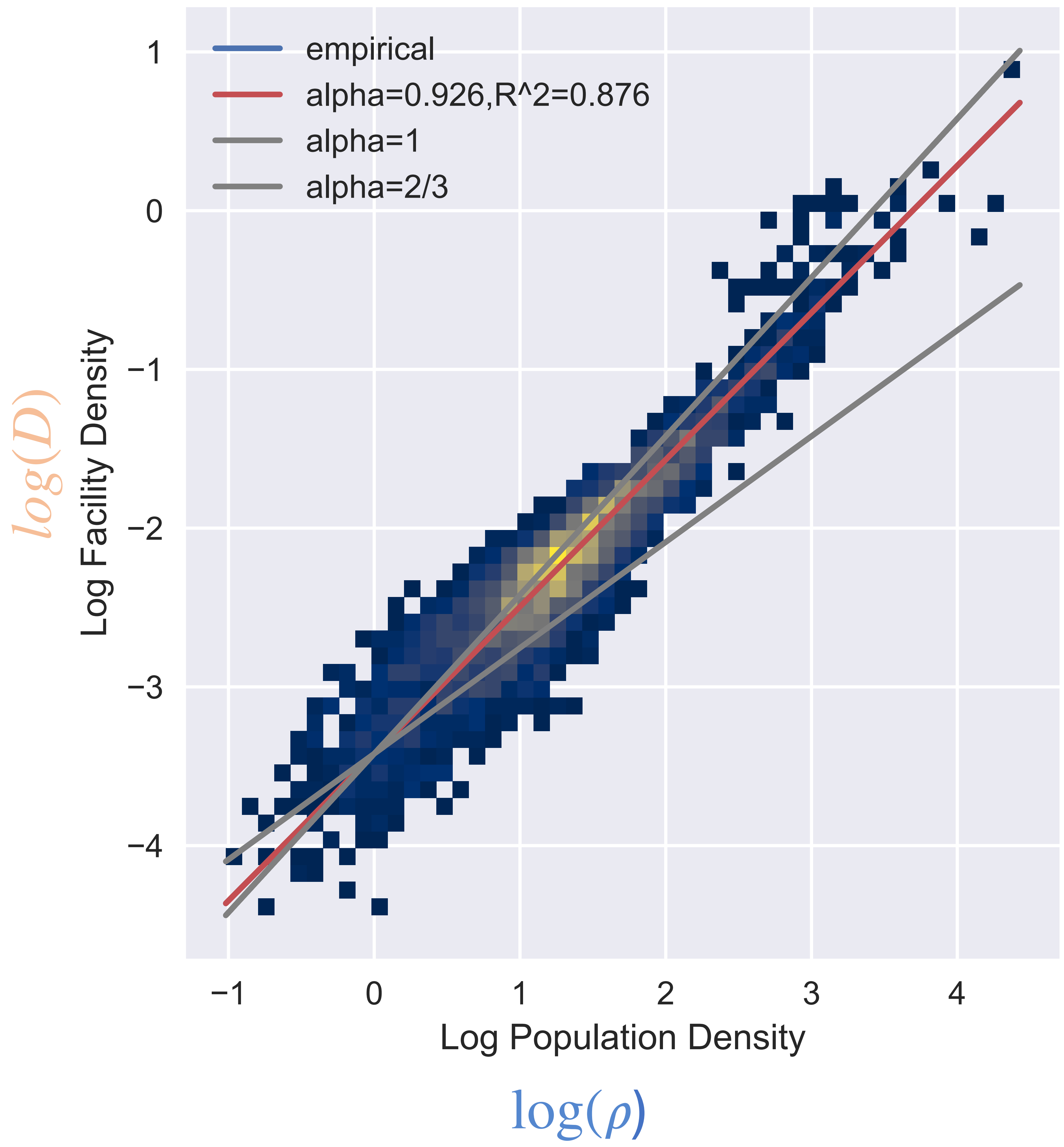
2. For each cell, calculate **population density**, ρ and **facility density** D



$$\rho = \frac{\text{num of people}}{\text{area}}$$

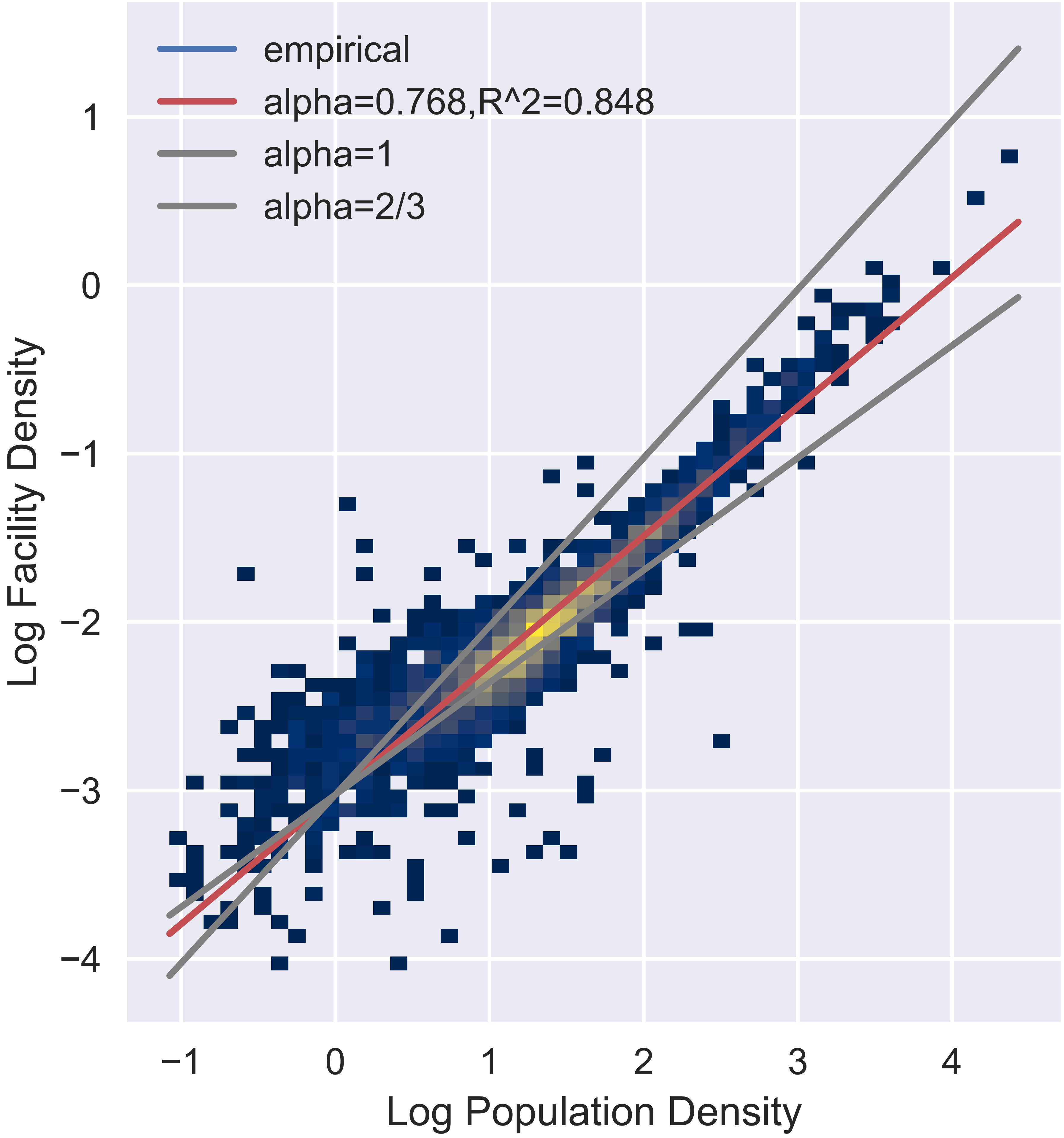
$$D = \frac{1}{\text{cell area}}$$

3. Fit data to a Reduced Major Axis(RMA) regression[7], slope is scaling exponent



Facility Data Scaling

Schools



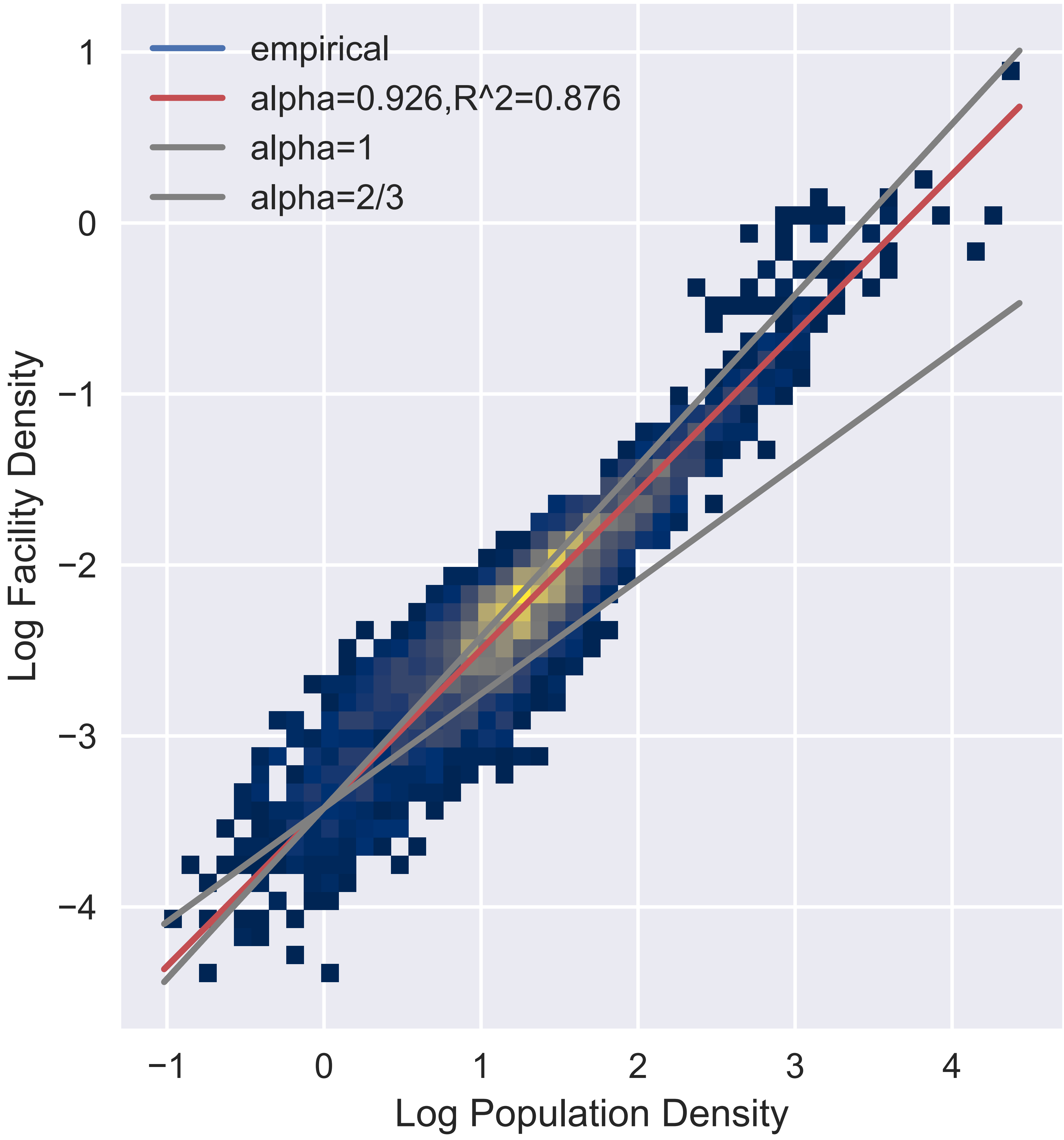
Scaling for Schools: 0.76

Public facilities should have an exponent close to 2/3

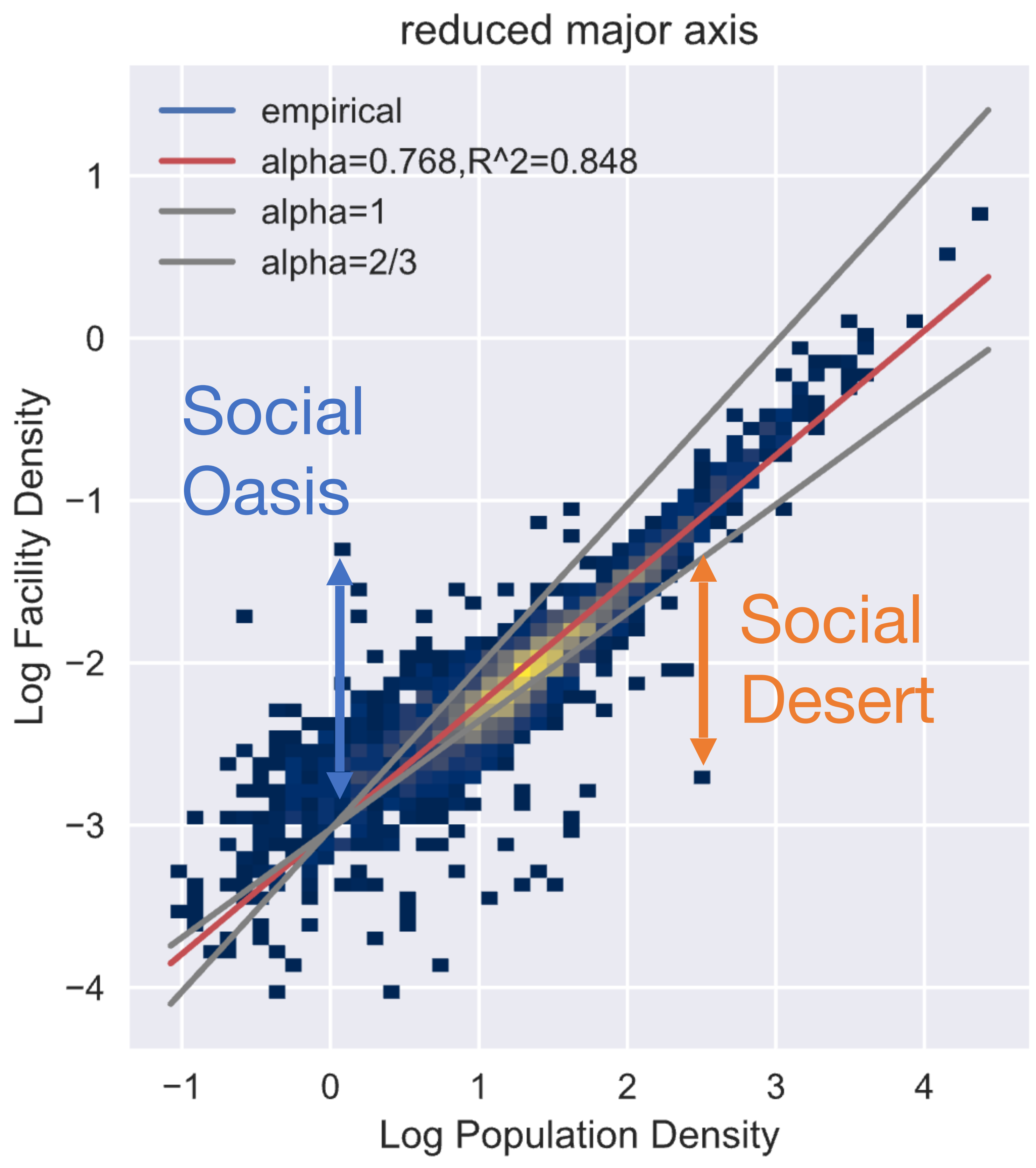
Private facilities should have an exponent close to 1

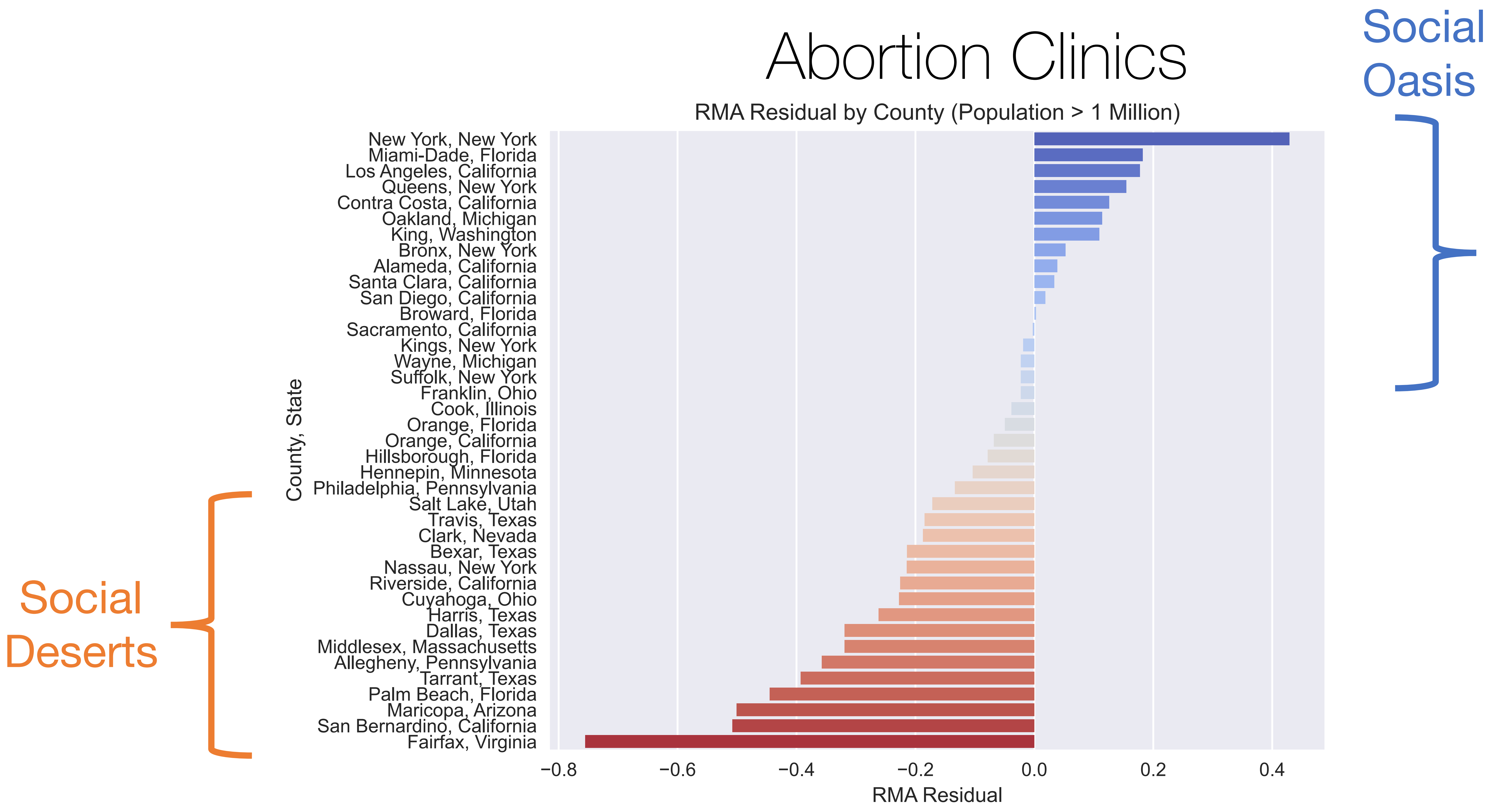
Scaling for Banks: 0.92

Banks

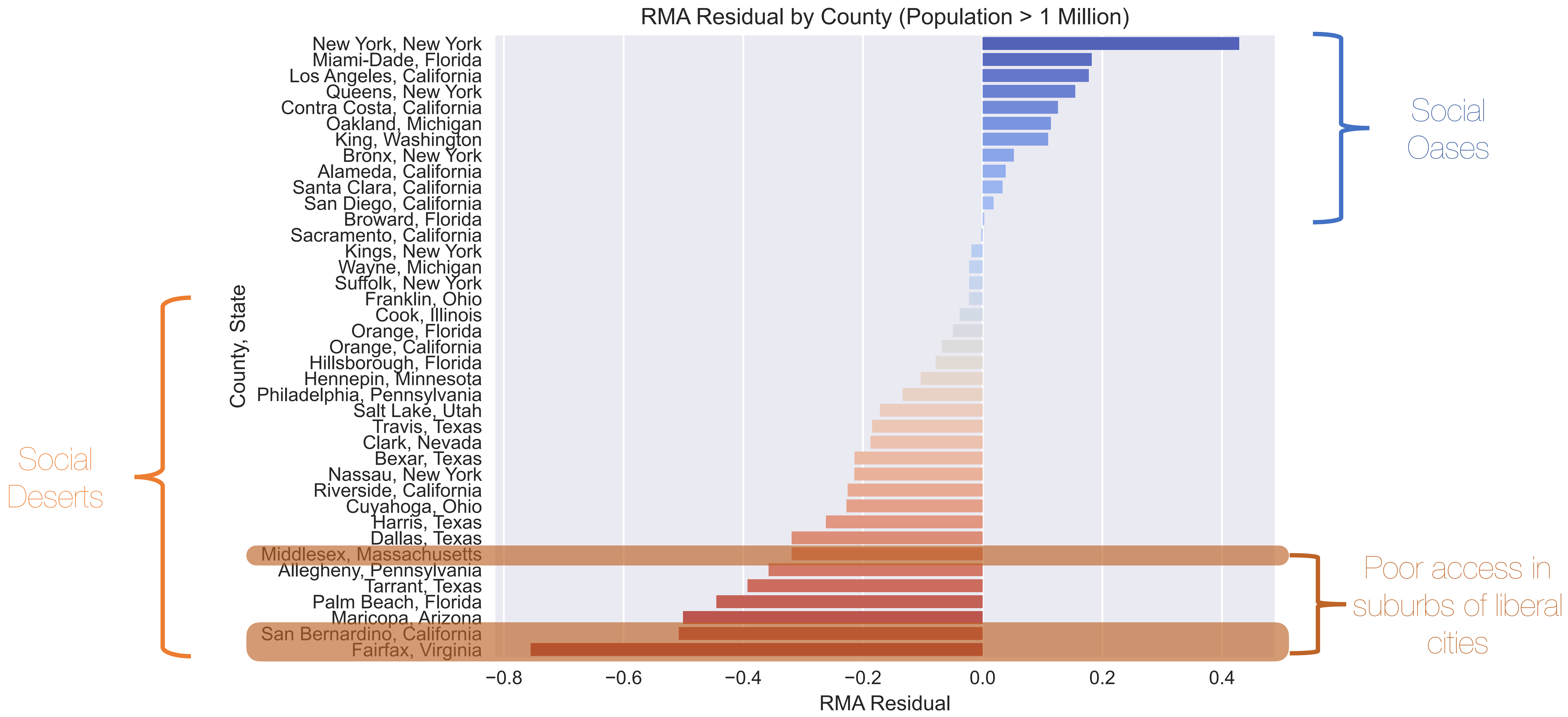


Residuals: Deserts and Oases



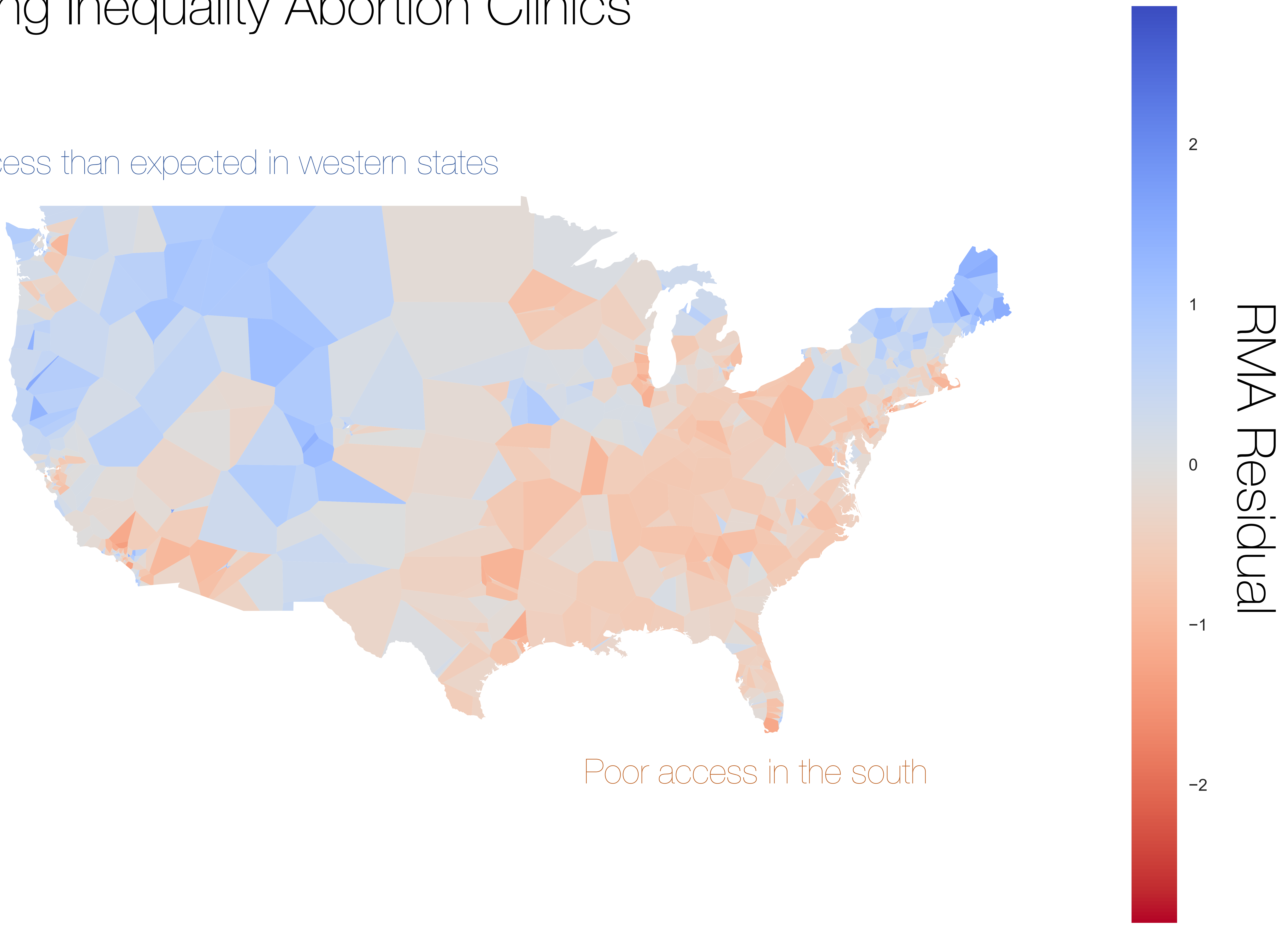


Abortion clinics access is not *simply* partisan



Mapping Inequality Abortion Clinics

More access than expected in western states

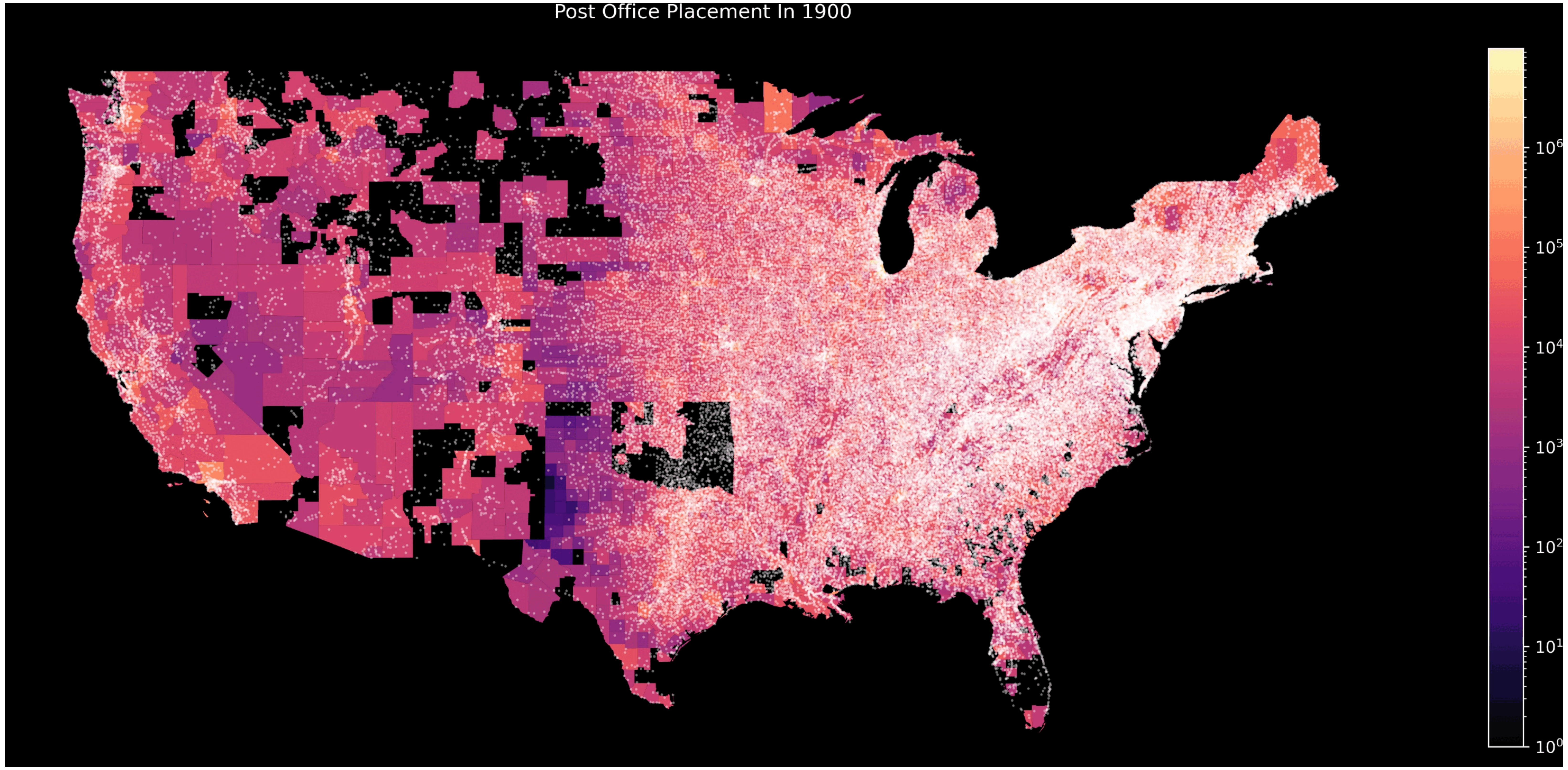


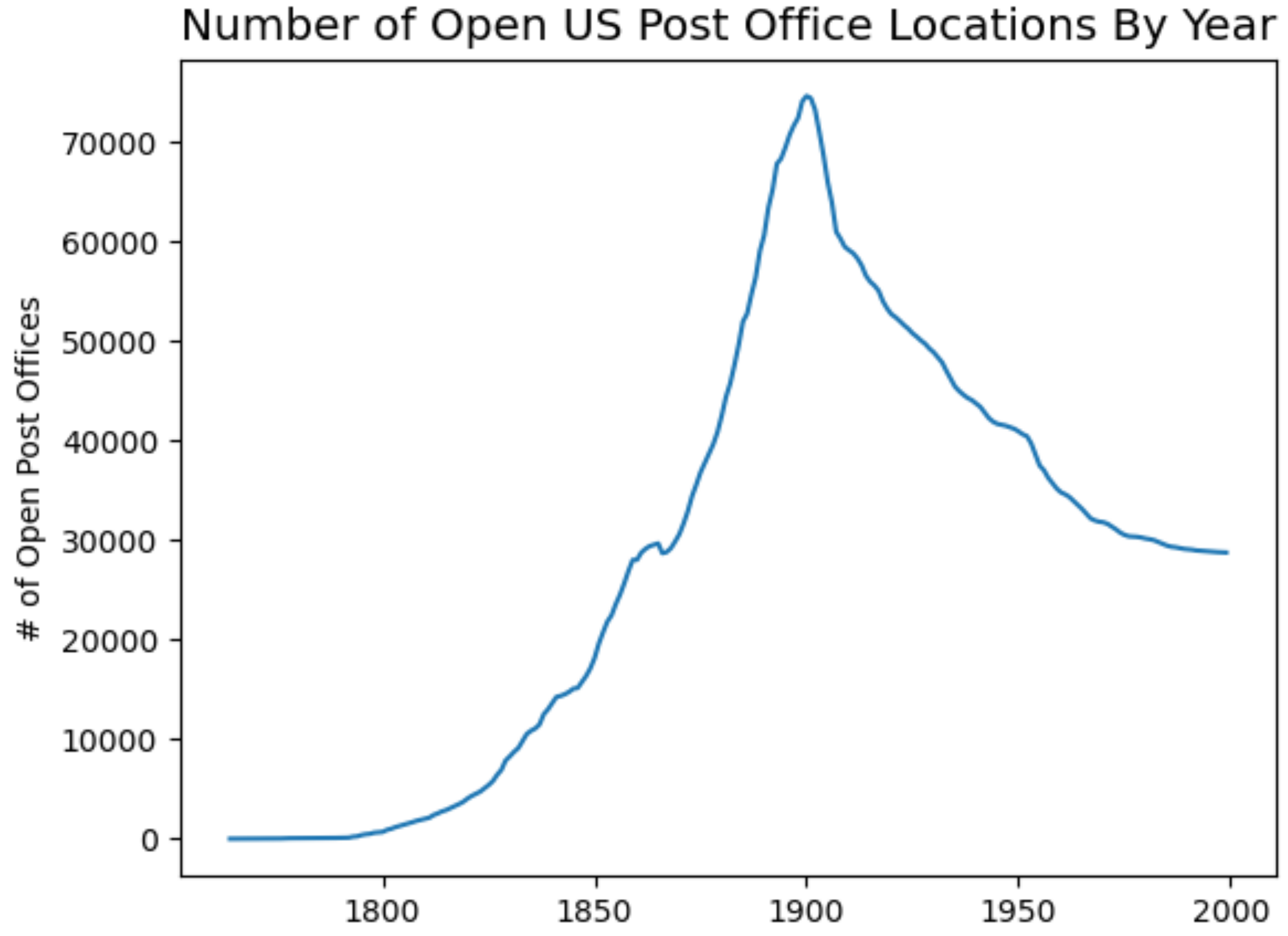
Poor access in the south

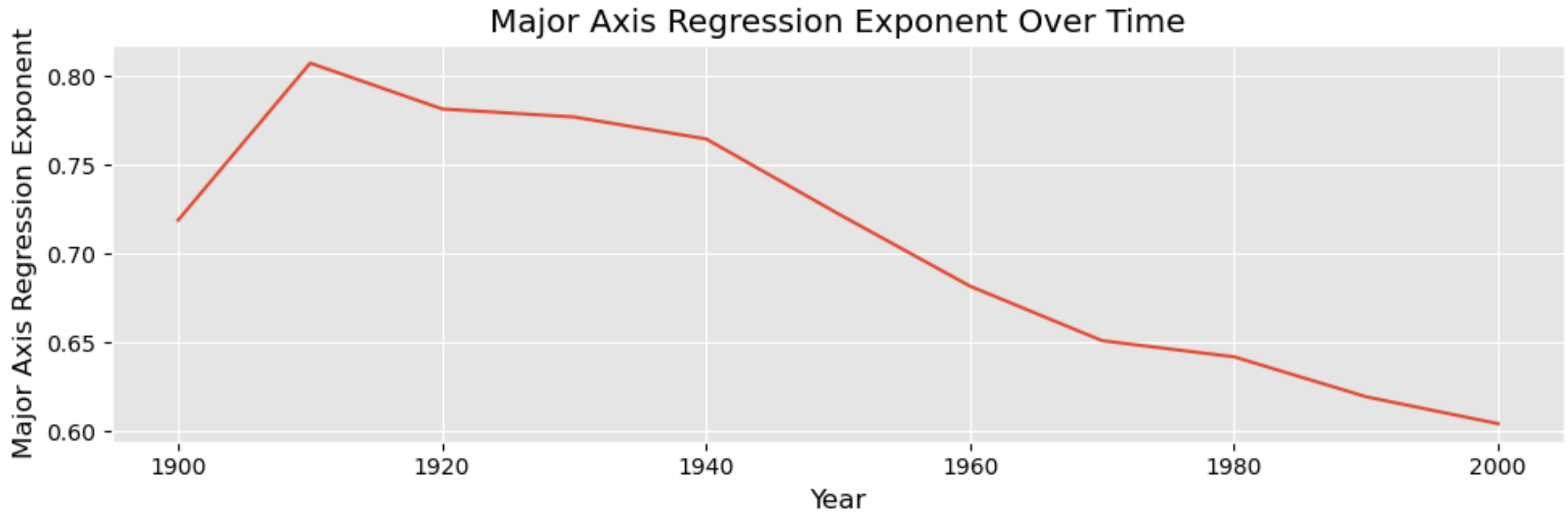
CHANGES IN DEMAND AND SUPPLY

Changes in Demand: Post Office Scaling

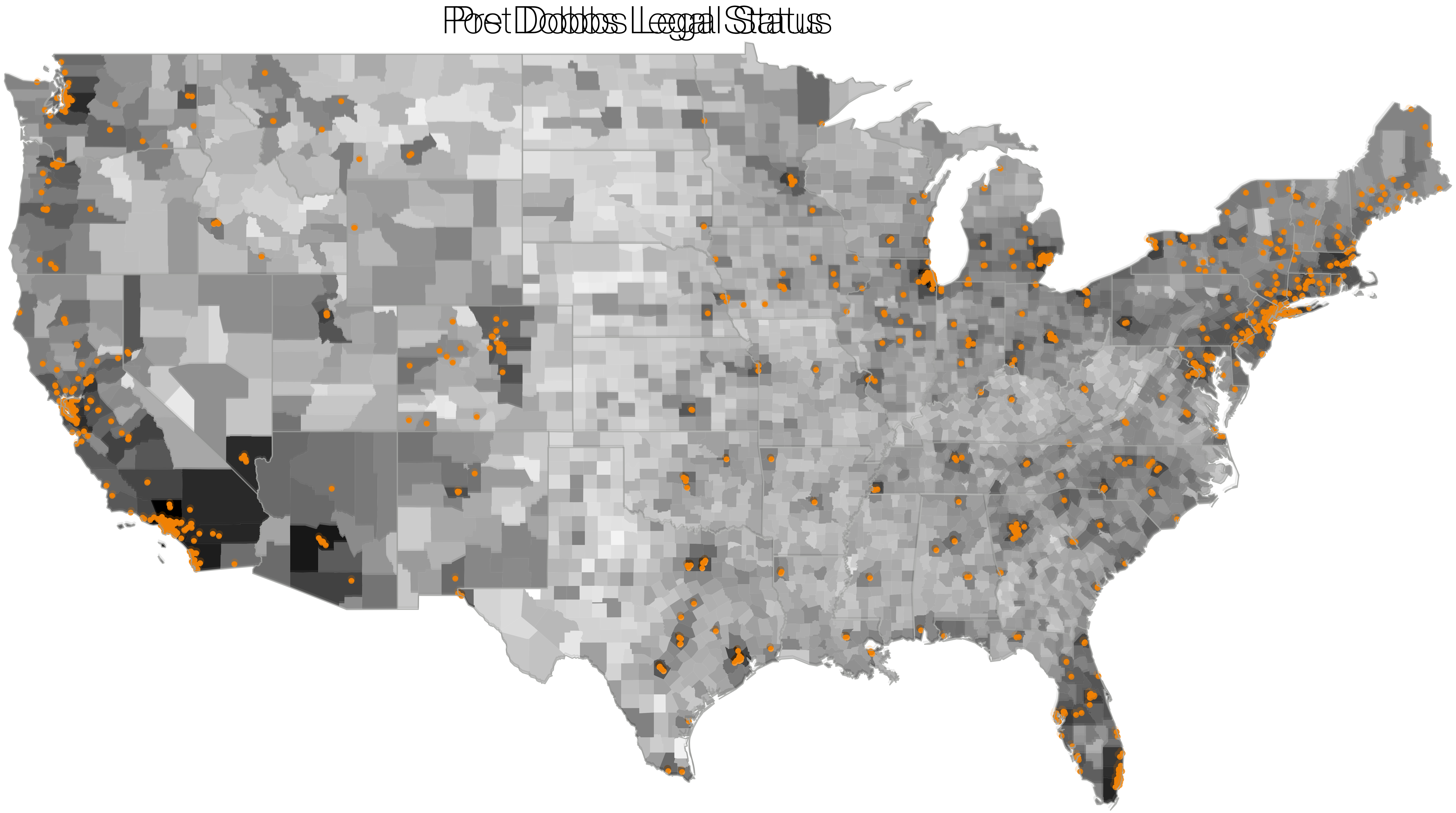
Post office scaling exponent has dropped from 0.8 to 0.6
commercial to public transition





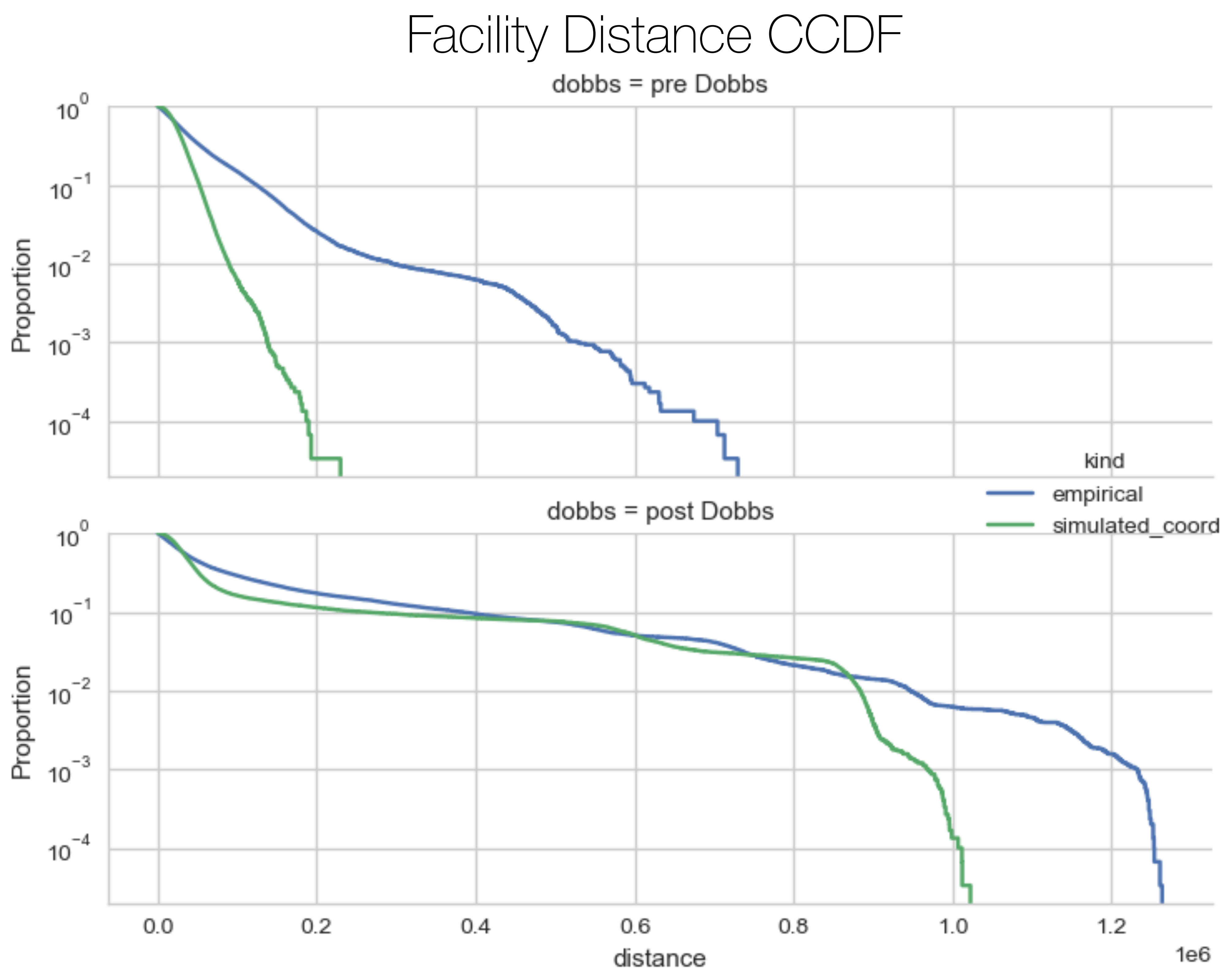


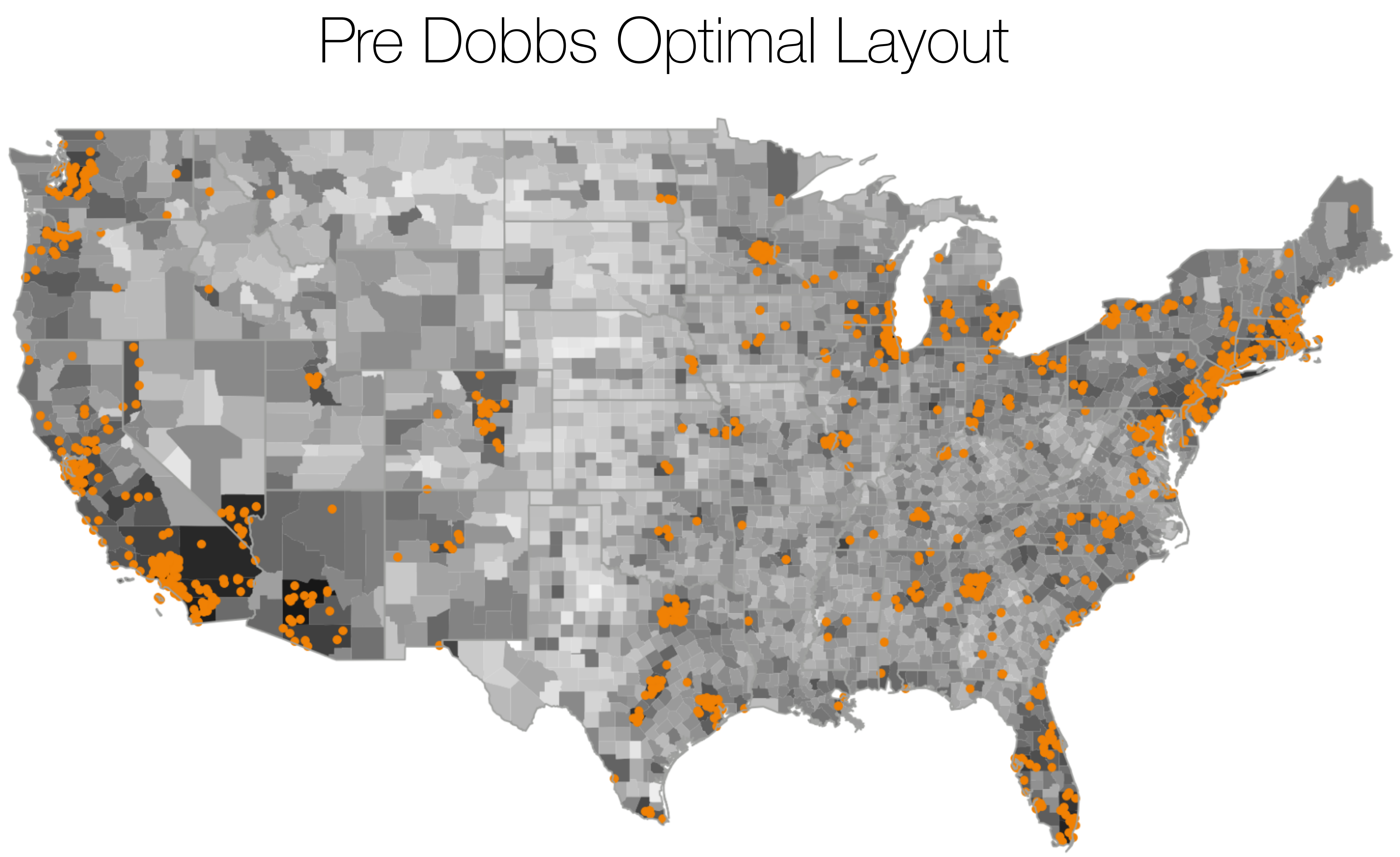
Changes in Supply:
Abortion Clinics

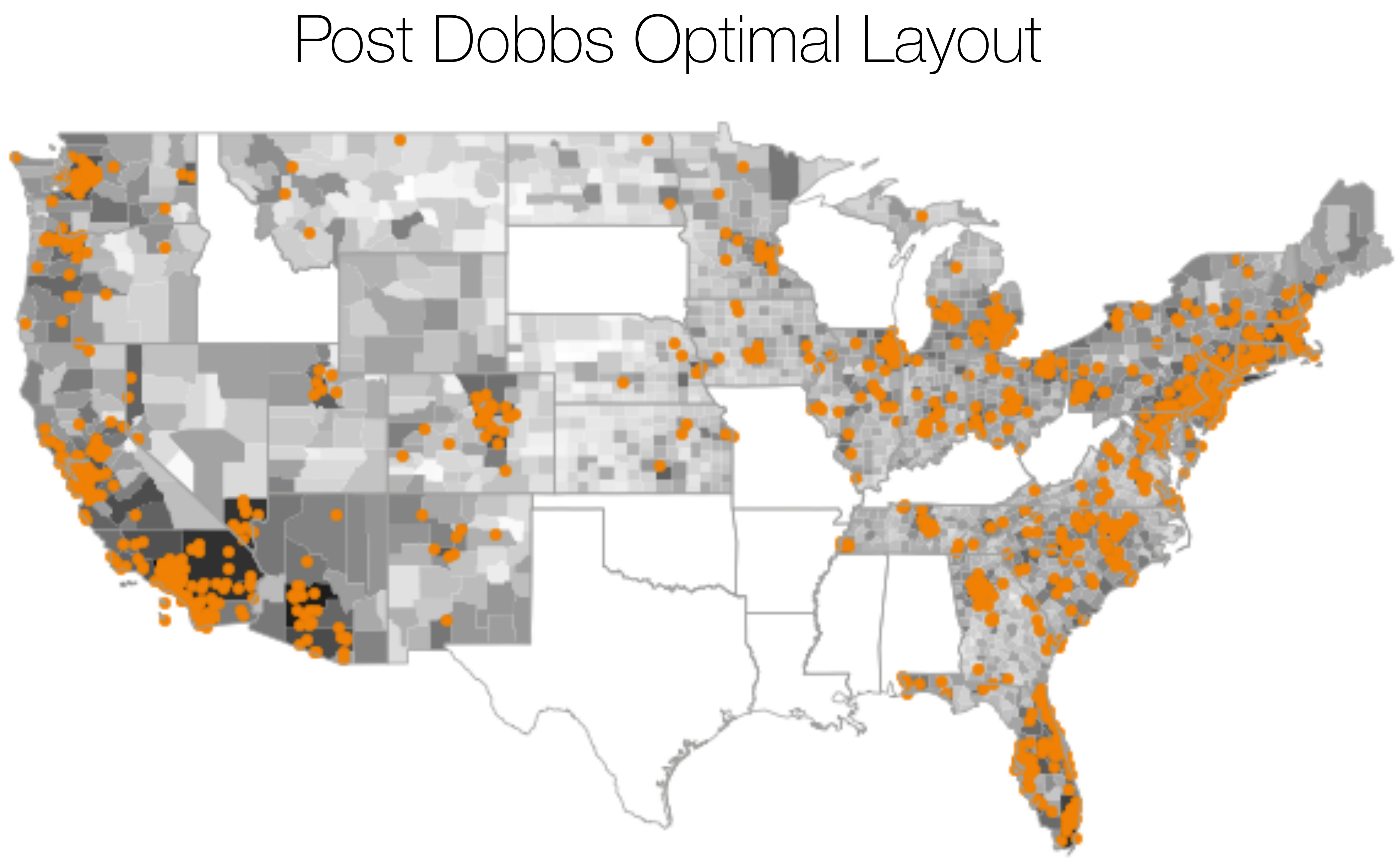


Changes in Supply: Abortion Clinics

How much does Dobbs change the travel distance to the nearest abortion clinic?
Compare empirical layout to optimal layout



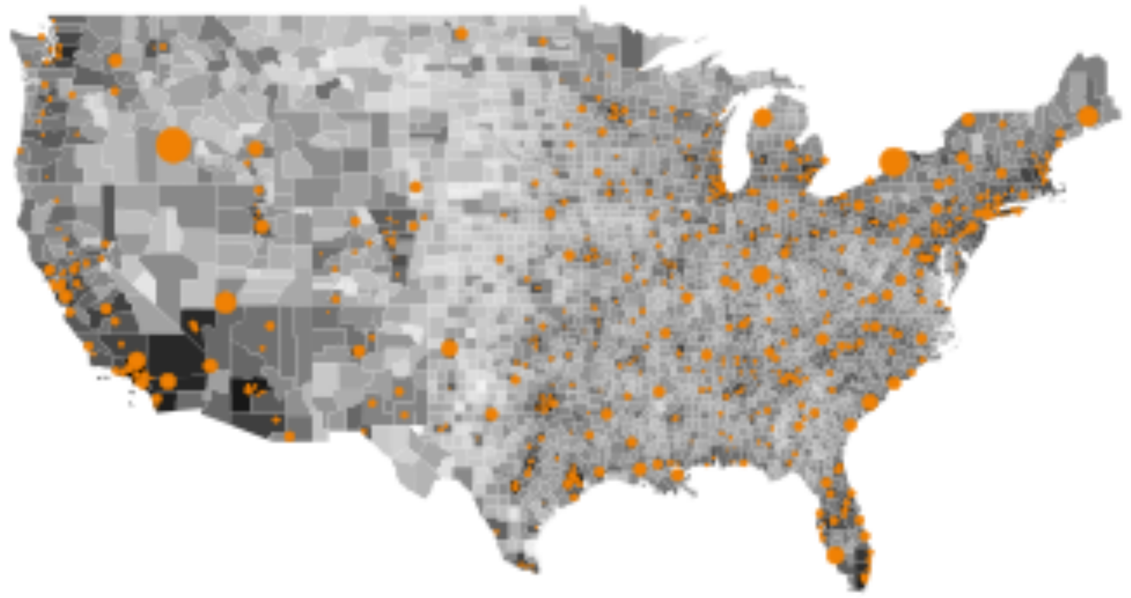


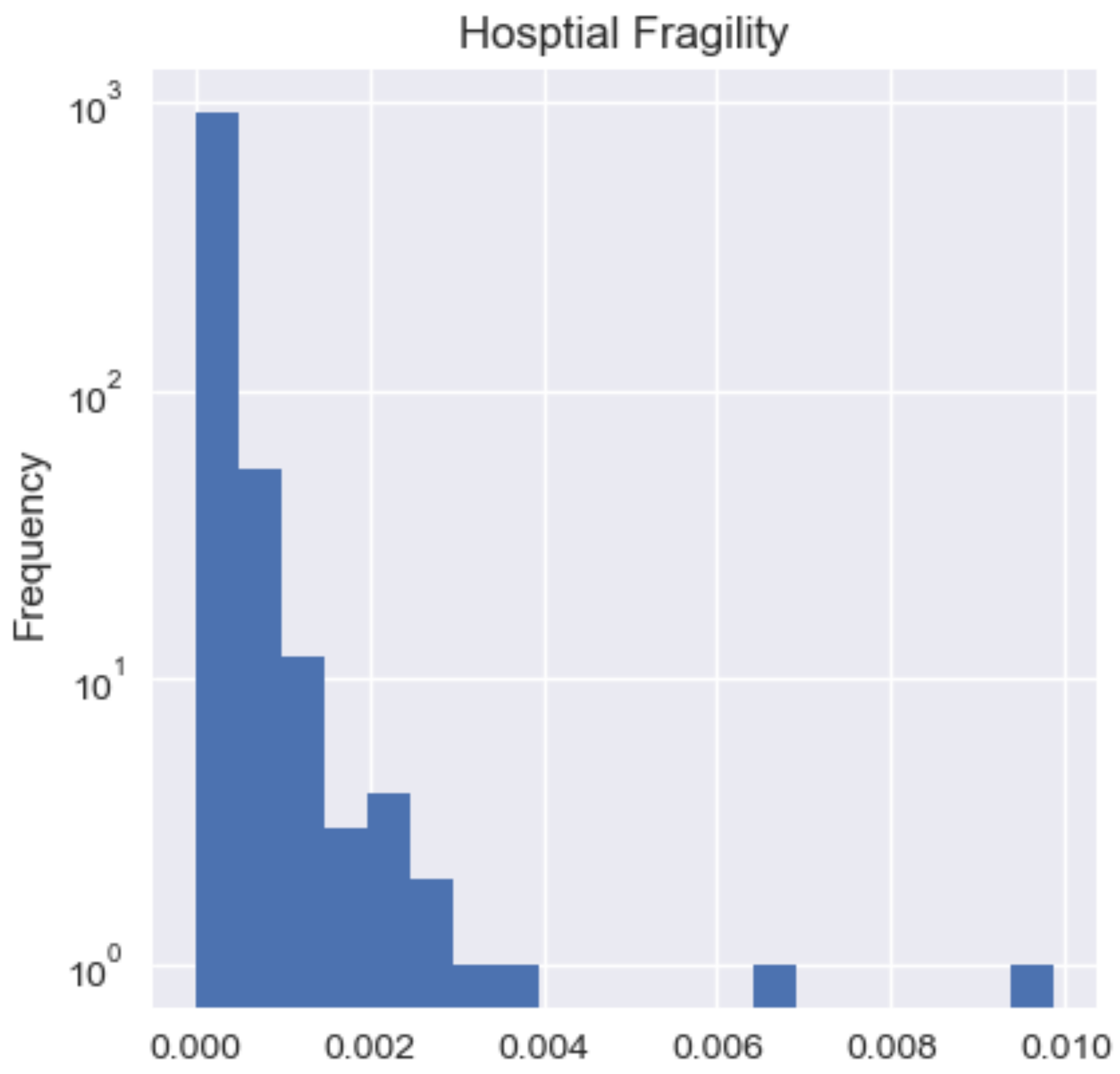


Fragility

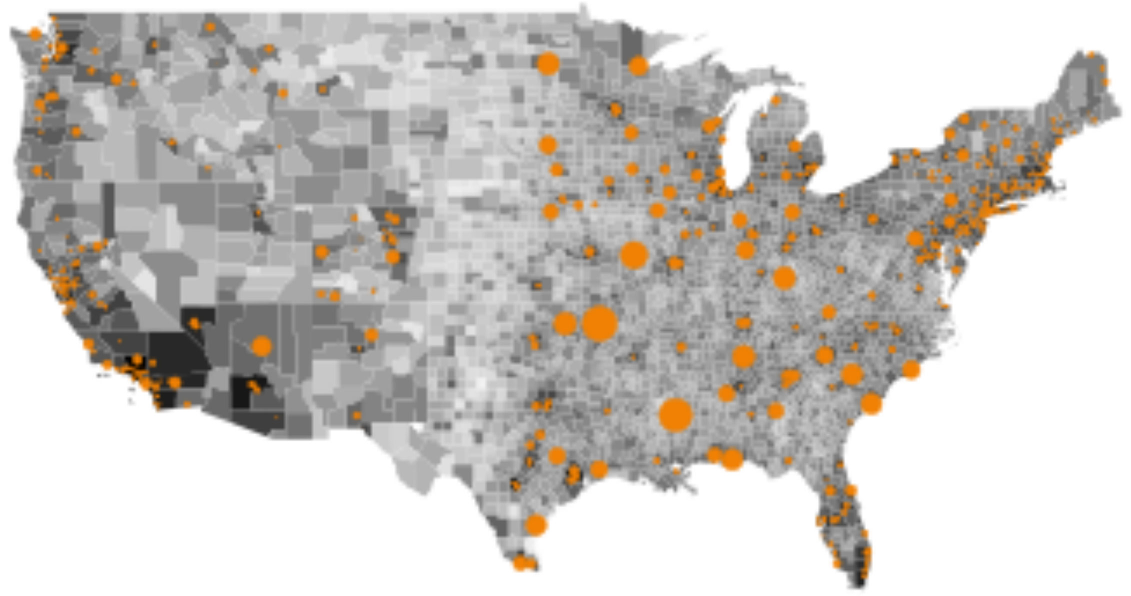
- Fragility: The fragility of facility i is the decrease in objective function caused by removing facility i from the genome
- Hospitals:
  - Boise ID, Buffalo NY, St George UT, Bangor ME
- Abortion Clinics:
  - Memphis TN, Little Rock AK, Jackson MI, Corpus Cristi TX, Las Vegas NV and Fargo, ND
- Conclusion:
  - Hospitals which are in urban centers in otherwise rural areas are fragile
  - Clinics which are in southern cities are the most fragile

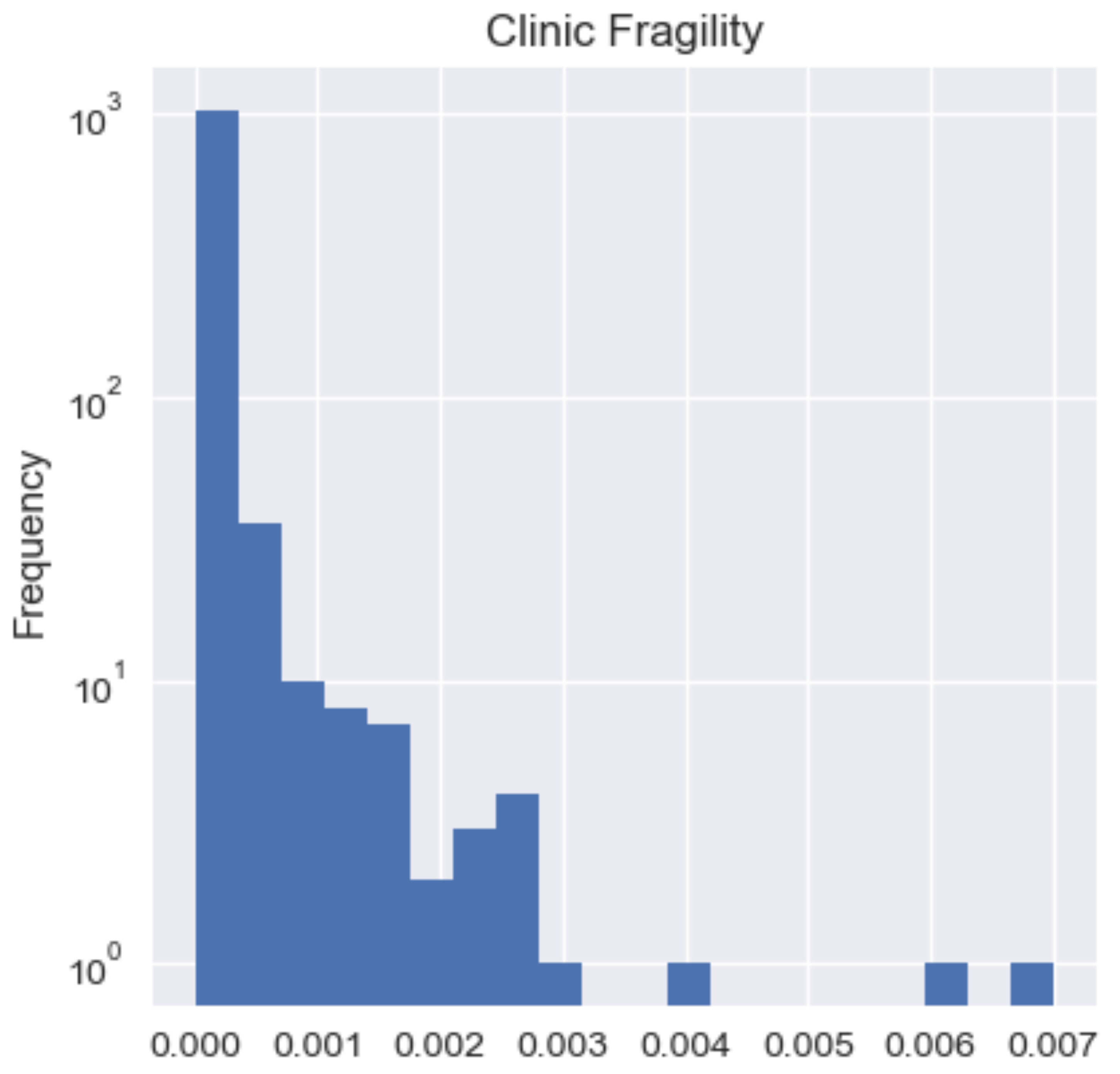
Hospitals





Abortion Clinics



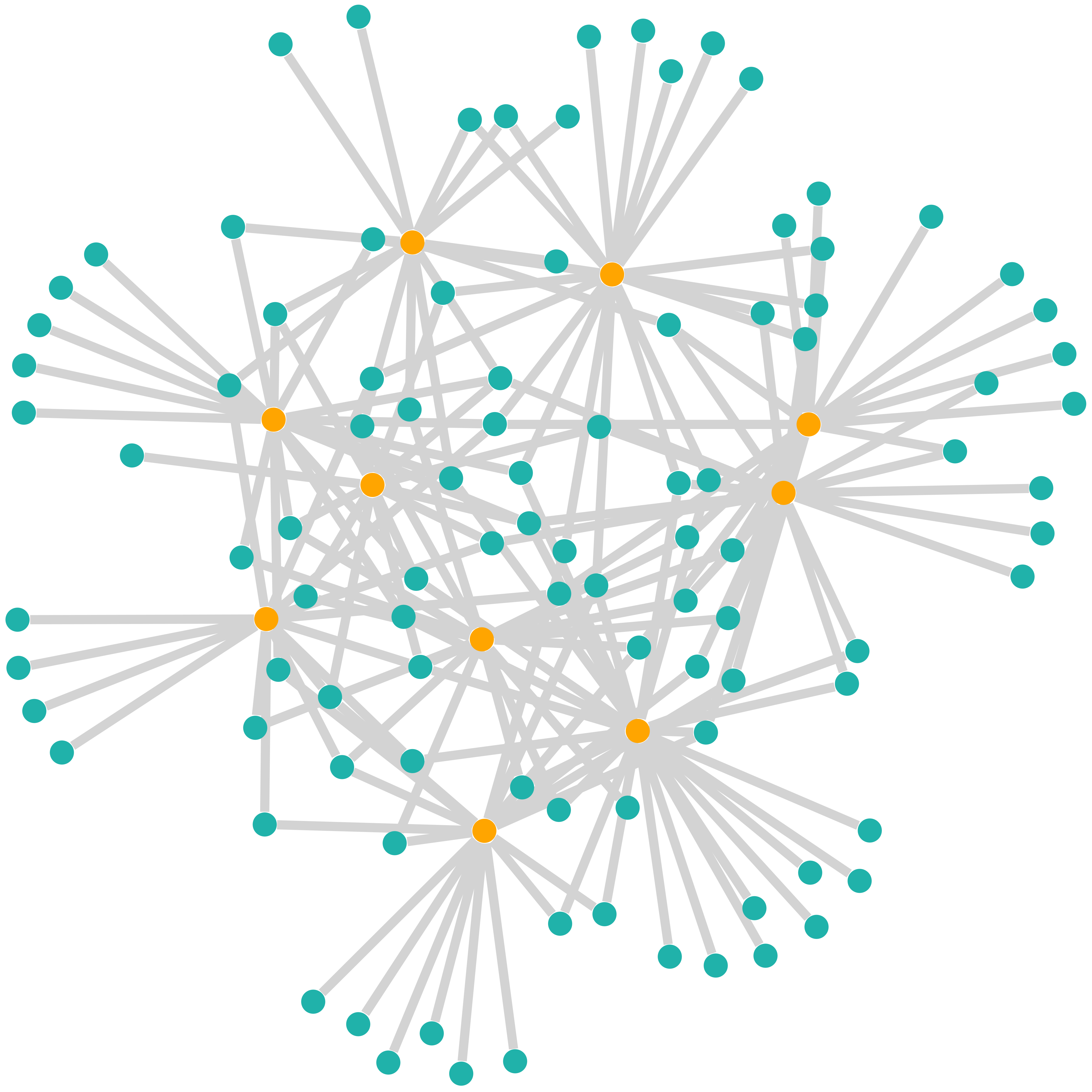


Conclusion

- Scaling relationships derived from optimal facility placement can be used to identify social deserts and social oases
- Changes in demand are detectable through scaling exponent changes
- Social deserts in aboriton clinic access don not fall along simply partisan lines

Part 2:

The Emergence of Polarization

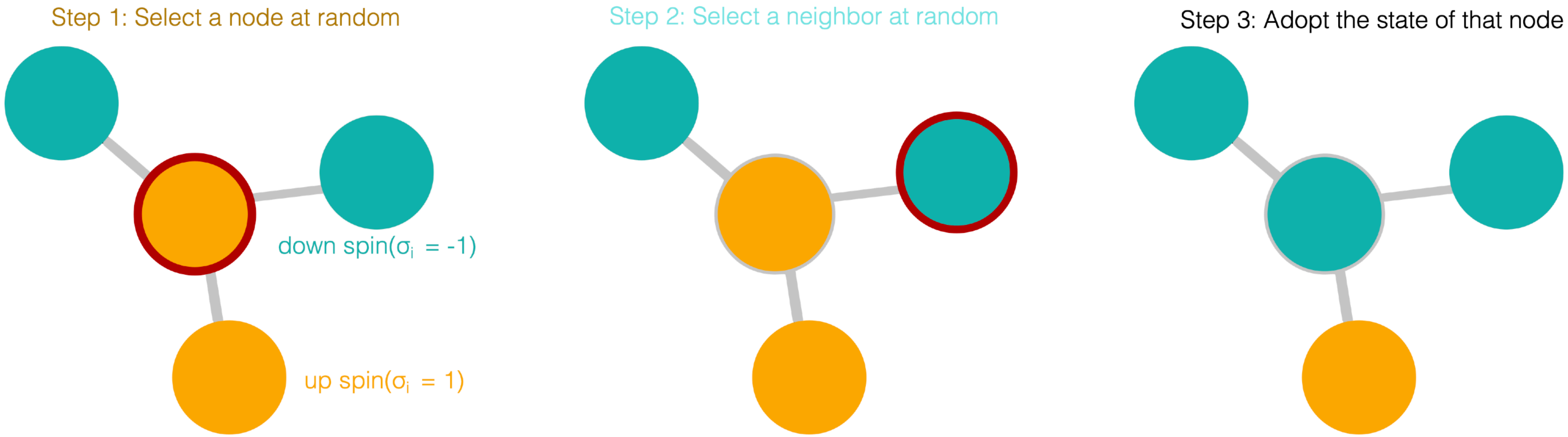


Groups Groups Groups

Social interactions are not pairwise - groups matter!

How do higher-order interactions affect the development of consensus?

Voter Models
Linear Voter Model



Voter Model Steps

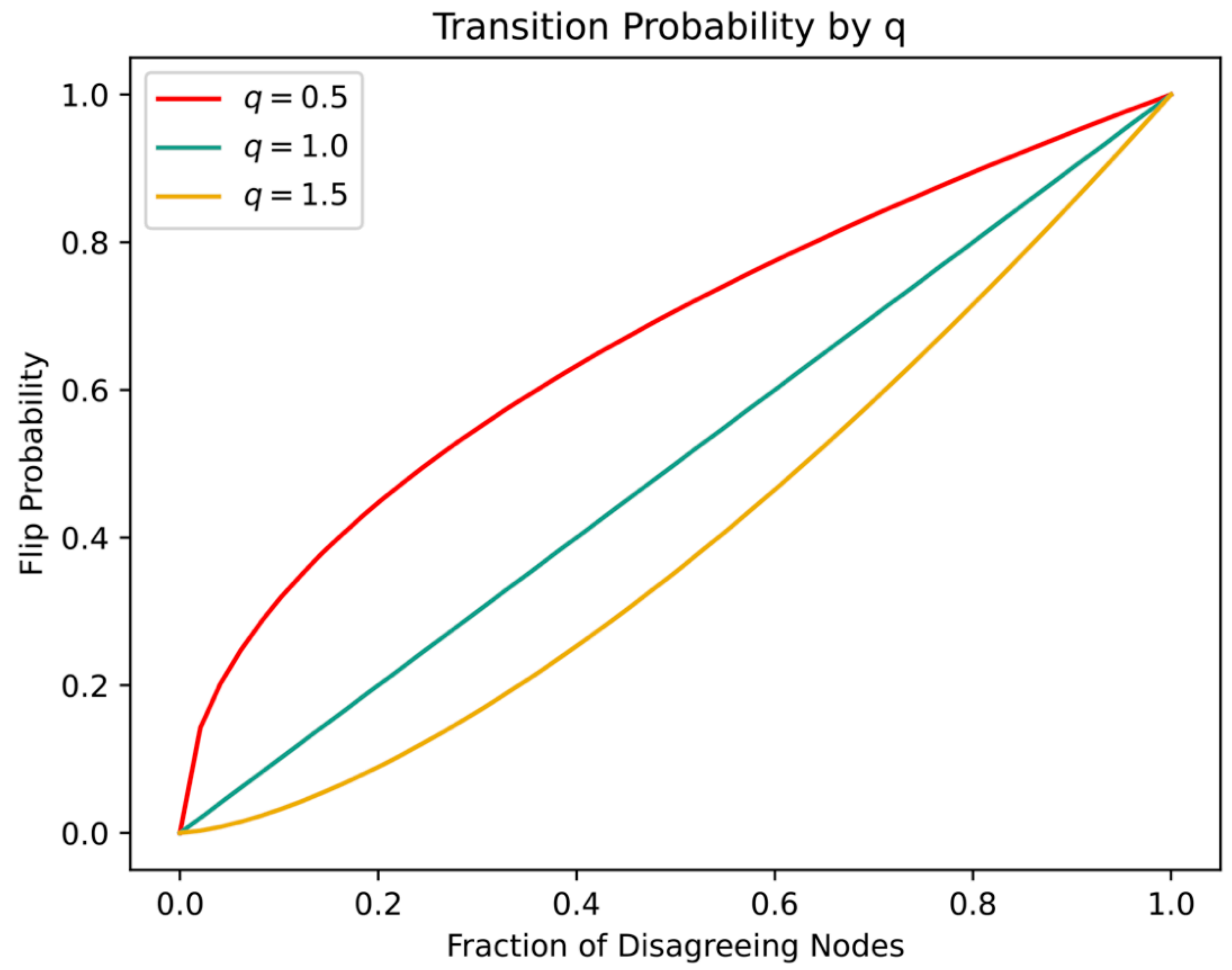
1. A random node i with state $\sigma_i \in \{-1, 1\}$, is selected
2. The selected node adopts the spin σ_j of a randomly selected neighbor $j \in \mathcal{N}_i$
3. Process is repeated until consensus is reached.
4. Transition rate for a node $\dot{\sigma_i} \propto \text{fraction of disagreeing neighbors}$

Voter Models

Non-Linear

Q Voter Model Steps[a]

1. A random node σ_i selects q of its neighbors. If all of its neighbors have the same spin, σ_i adopts that spin
2. Transition rate for a node
   $\dot{\sigma}_i \propto \text{fraction of disagreeing neighbors}^q$

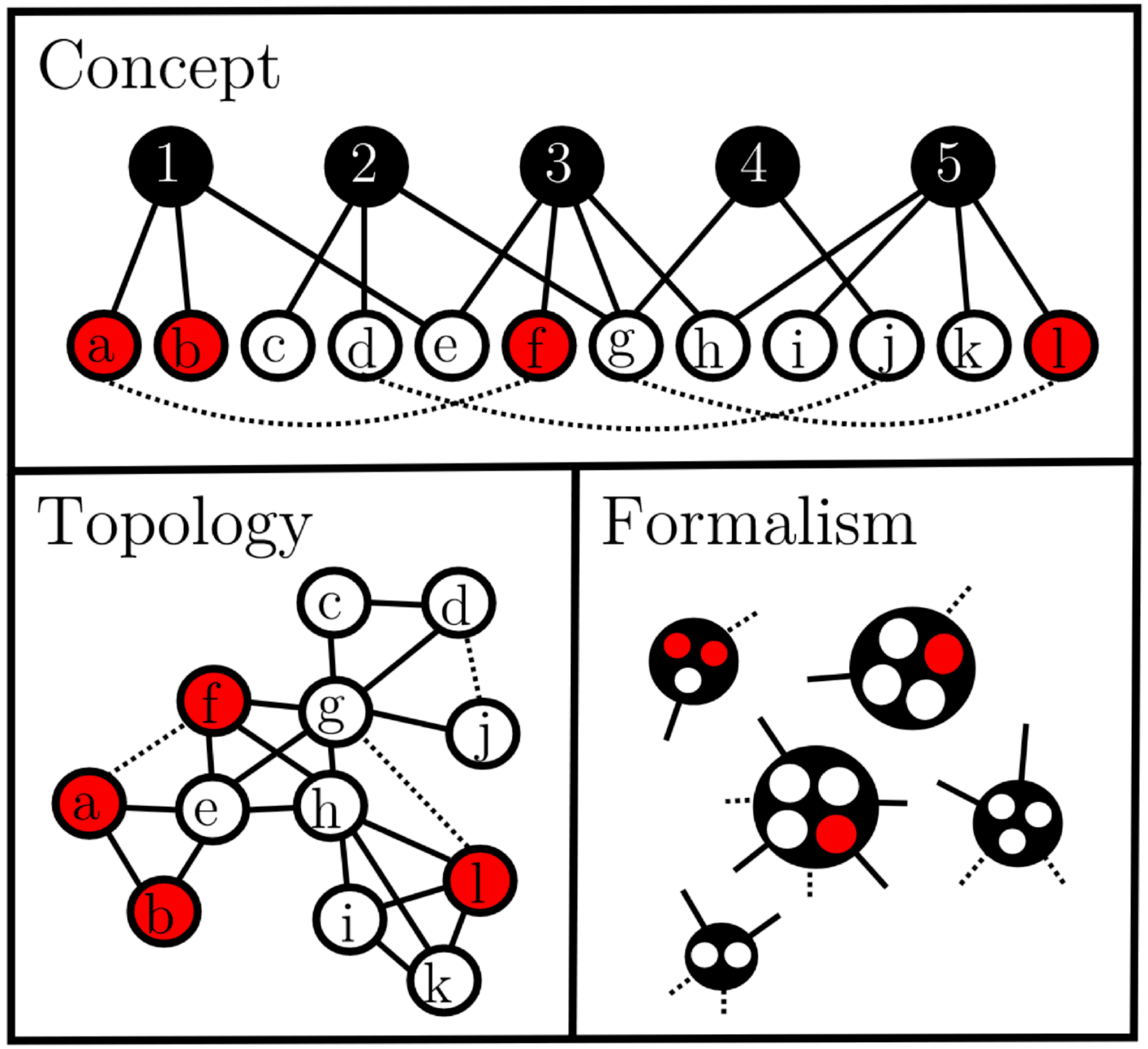


What does q do?

- q controls the conformity bias of the model.
- if $q > 1$: conformist nodes nodes, if $q < 1$, we get the hipster nodes nodes.

[a] Castellano et al., 2009.

- Each node belongs to a set of cliques. Nodes interact with other nodes in the same clique.
- Higher-order network topology generated by the model proposed by Newman[a].



Description

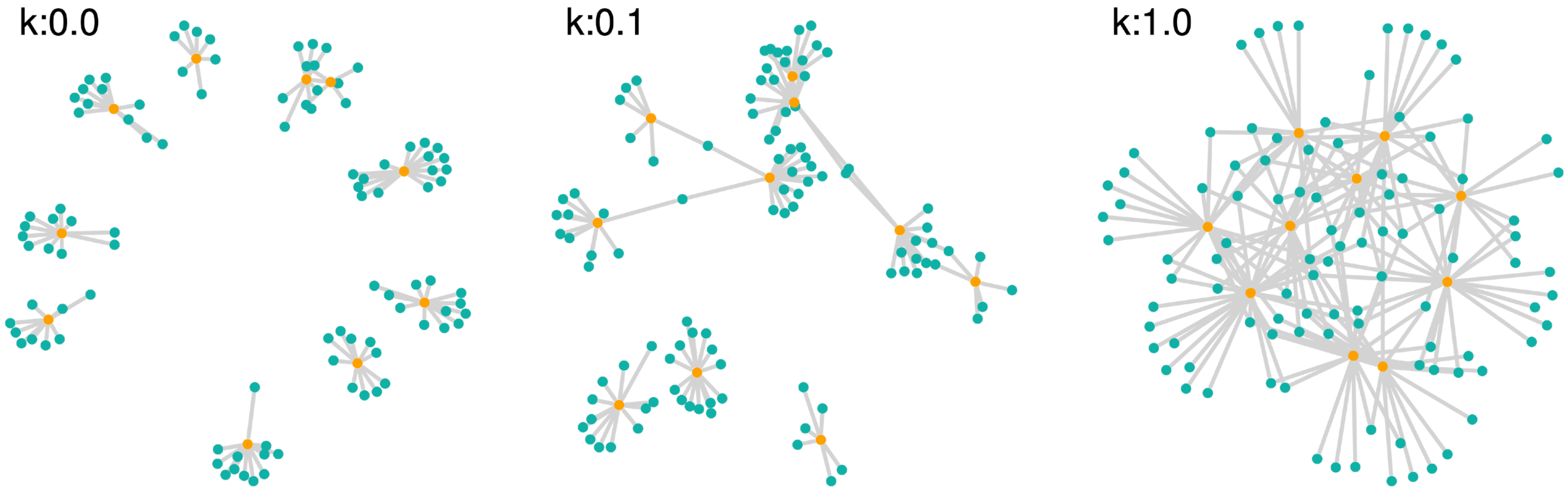
The model is parameterized by two distributions

1. N the number of nodes
2. M the number of cliques
3. $\{p_n\}$ the distribution of nodes per clique
4. $\{g_m\}$ the distribution of cliques per node

[a] Newman, 2003.

Clique Coupling

$\langle k_{ex} \rangle$ determines the coupling between groups



Deriving the Master Equation

The First Term

- Approximate master equations(AMES) are high accuracy approximations of binary state dynamics on networks[a]
- Occupation number :
  G_u the fraction of the system in a clique with u up spins.
- Example: up spin out flux : the rate at which down spins flip to up spins

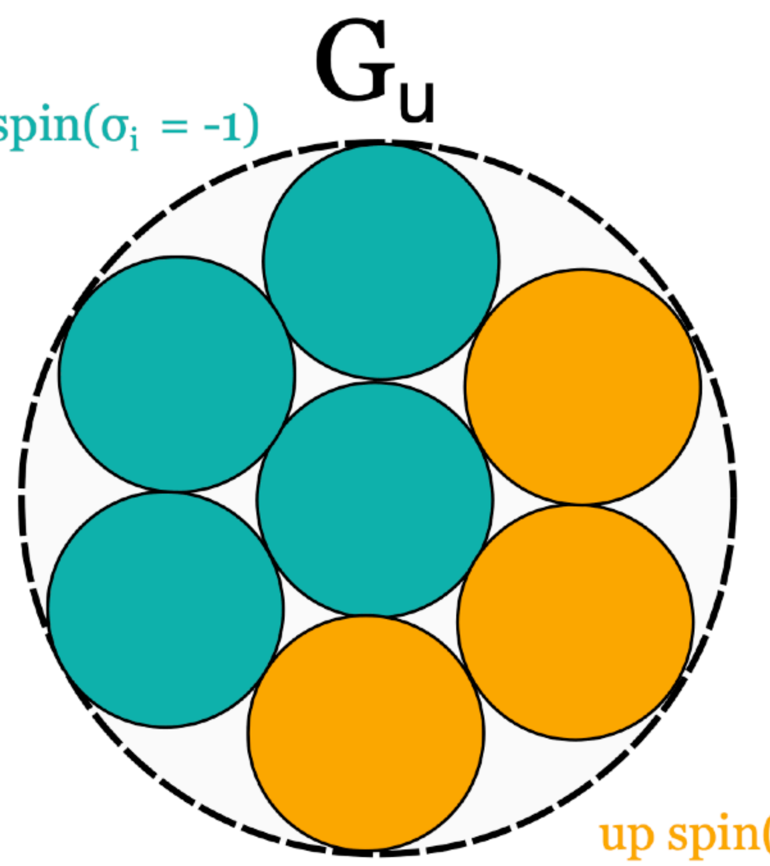
occupation number

fraction of up spins in clique

$$P(G_u \to G_{u+1}) = G_u \; (n-u) \left(\frac{u}{n}\right)^q$$

up spin out flux

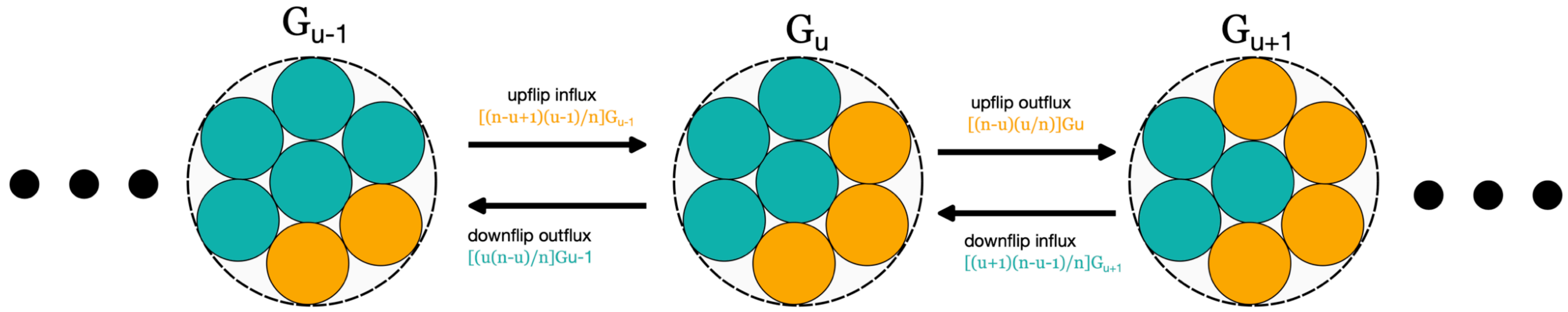
of down spins in clique



[a] Gleeson, 2011; Hébert-Dufresne et al., 2010; St-Onge et al., 2021.

Deriving the Master Equation

The Whole Shebang



Definition 3.1

Voter Model Master Equation for Constant Clique Size with Uncoupled Cliques

up spin in flux — *down spin in flux*

$$\frac{dG_u}{dt} = G_{u-1}\left[(n-u+1)\left(\frac{u-1}{n}\right)^q\right] + G_{u+1}\left[(u+1)\left(\frac{n-u-1}{n}\right)^q\right] -$$

$$G_u\left[(n-u)\left(\frac{u}{n}\right)^q\right] - G_u\left[(u)\left(\frac{n-u}{n}\right)^q\right]$$

(1)

up spin out flux — *down spin out flux*

Definition 4.1

Moment Closure
The moment closure approximates the coupling between a group and surrounding groups

$$\rho_u(t) = \langle k_{ex} \rangle \frac{\sum_u G_u((n-u)\left(\frac{u}{n}\right)^q)}{\sum_u G_u(n-u)} \tag{2}$$

$$\rho_d(t) = \langle k_{ex} \rangle \frac{\sum_u G_u(u\left(\frac{n-u}{n}\right)^q)}{\sum_u G_u(u)} \tag{3}$$

Definition 4.2

Voter Model Master Equation for Constant Clique Size and Moment Closure

$$\frac{dG_u}{dt} = G_{u-1}\left[(n-u+1)\left(\frac{u-1}{n}\right)^q + \rho_u\right] + G_{u+1}\left[(u+1)\left(\frac{n-u-1}{n}\right)^q + \rho_d\right] - G_u\left[(n-u)\left(\frac{u}{n}\right)^q + \rho_u\right] - G_u\left[(u)\left(\frac{n-u}{n}\right)^q + \rho_d\right] \tag{4}$$

Solving For The Steady State

Definition 5.1

Detailed Balance In equilibrium, each elementary process is in equilibrium with its reverse process.

$$P(G_u \to G_{u+1}) = P(G_{u+1} \to G_u)$$

$$P(G_u \to G_{u-1}) = P(G_{u-1} \to G_u)$$

We know the recursion formula is

$$G_u = \frac{(n-u+1)\left[\rho + \left(\frac{u-1}{n}\right)^q\right]}{u[\rho + \frac{n-u}{n}]} G_{u-1}$$

So the formula for G_u is

$$G_u = \frac{1}{Z} \prod_{i=0}^{u} \frac{(n-u+1)\left[\rho + \left(\frac{u-1}{n}\right)\right]}{u[\rho + \frac{n-u}{n}]}$$

Where

$$Z = \sum_{u=1}^{N} \prod_{j=0}^{u} \frac{(n-j+1)\left[\rho + \left(\frac{j-1}{n}\right)\right]}{j[\rho + \left(\frac{j-1}{n}\right)]}$$

LINEAR RESULTS

LINEAR VOTER MODEL(Q = 1)

Coexistence emerges as coupling increases

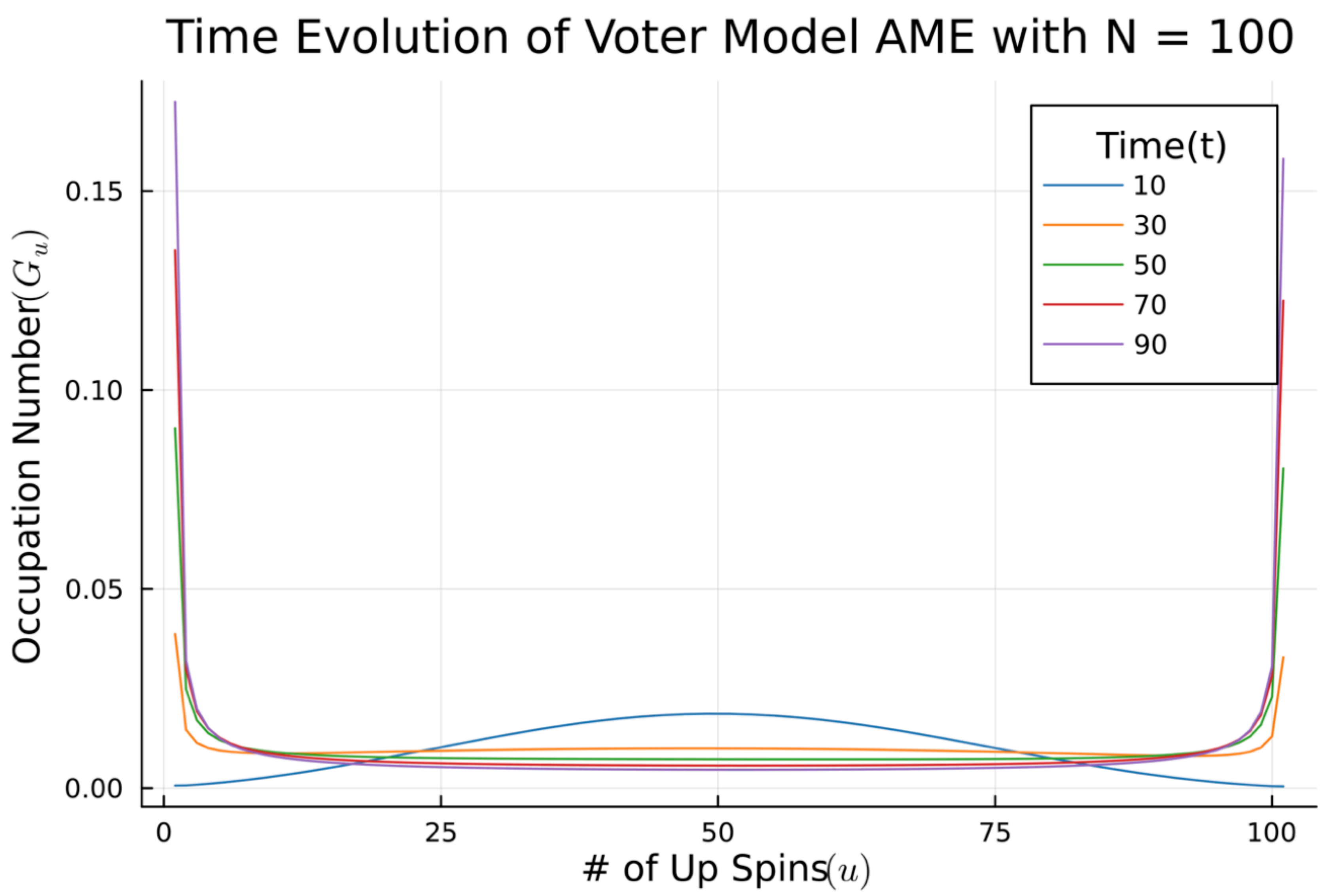


Figure. Time series of numerical integration of AME with $\rho = 0.0$ the distribution collapses two the two absorbing states

Coexistence emerges as coupling increases

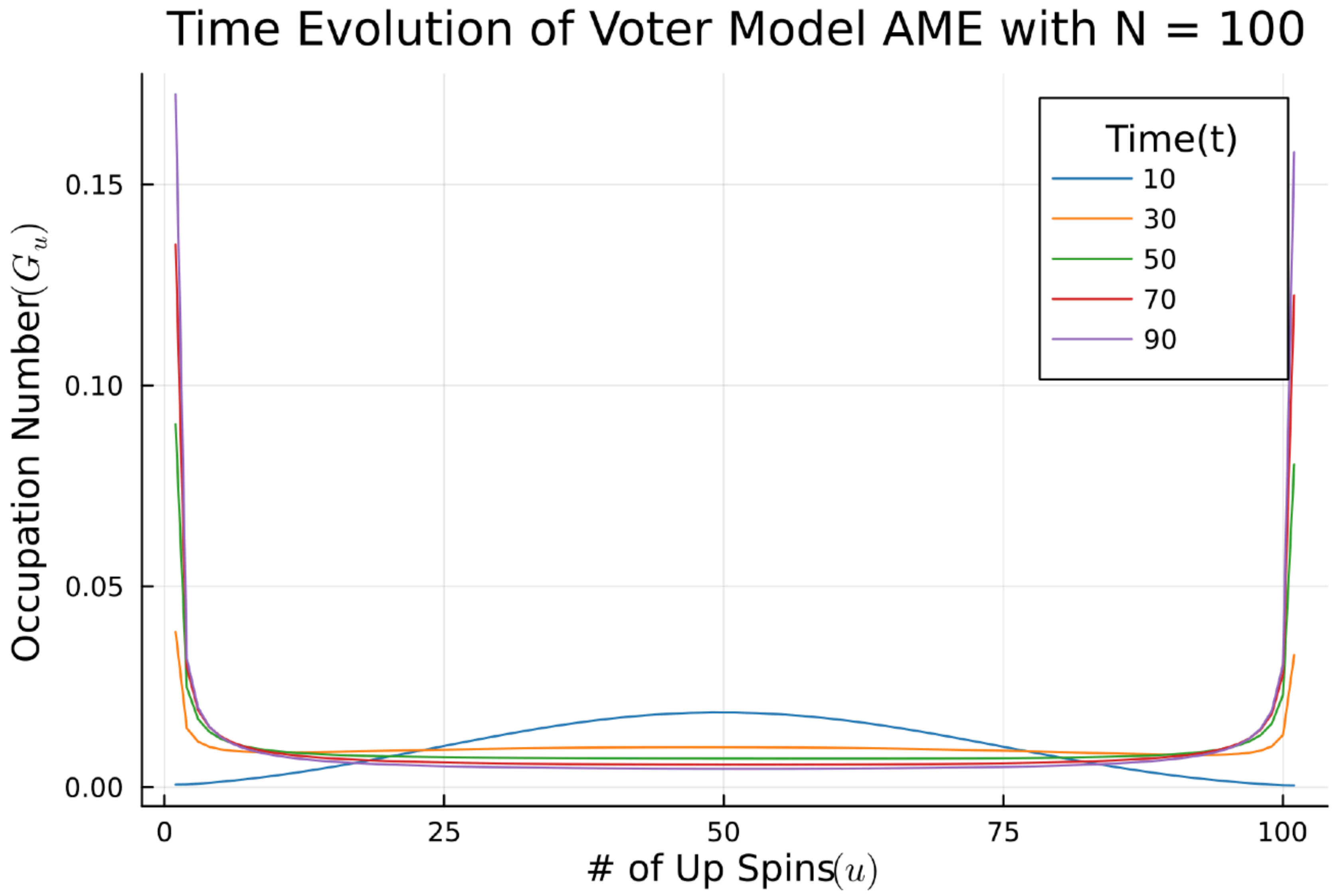


Figure. Time series of numerical integration of AME with $\rho = 0.0$ the distribution collapses two the two absorbing states

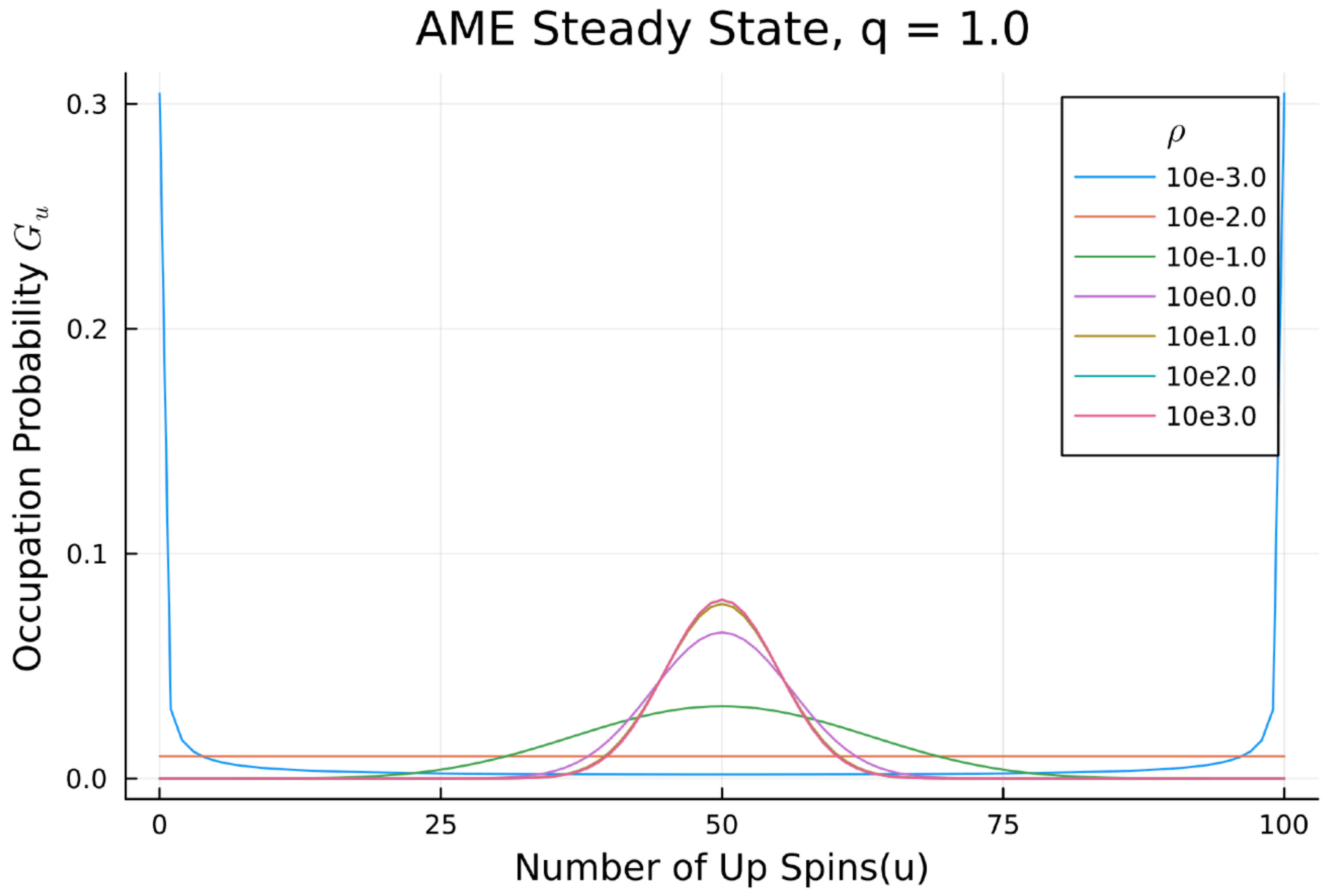
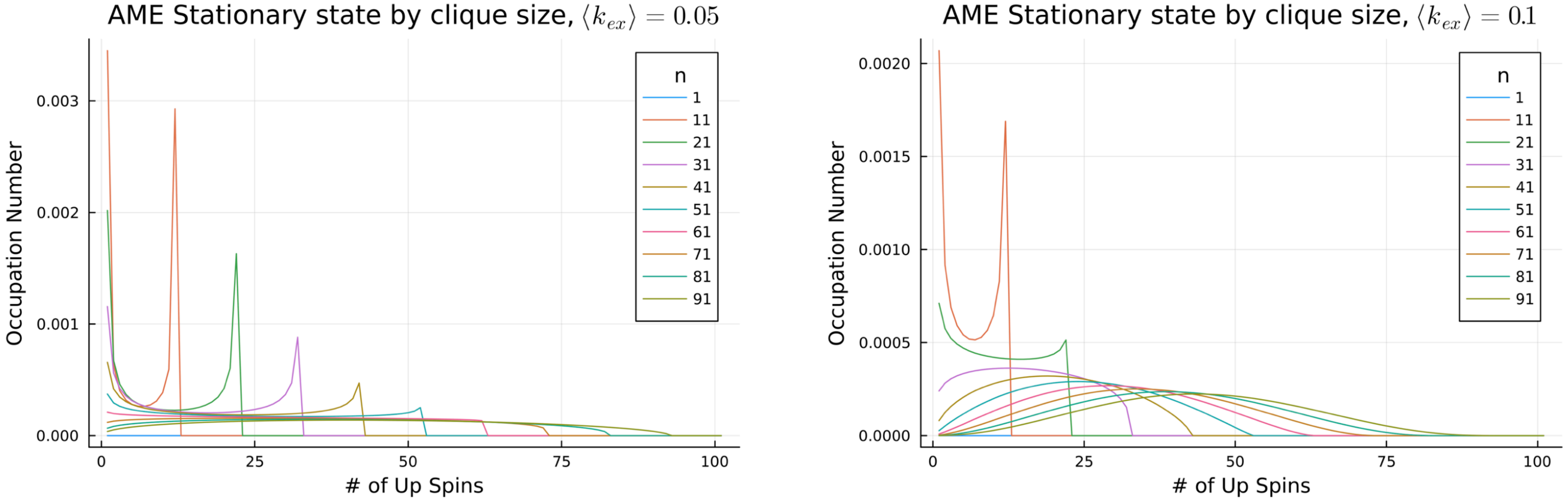


Figure. Steady state distribution for AMES as a function of ρ. Coexistence emerges as coupling increases

Larger cliques support coexistence at lower coupling strengths

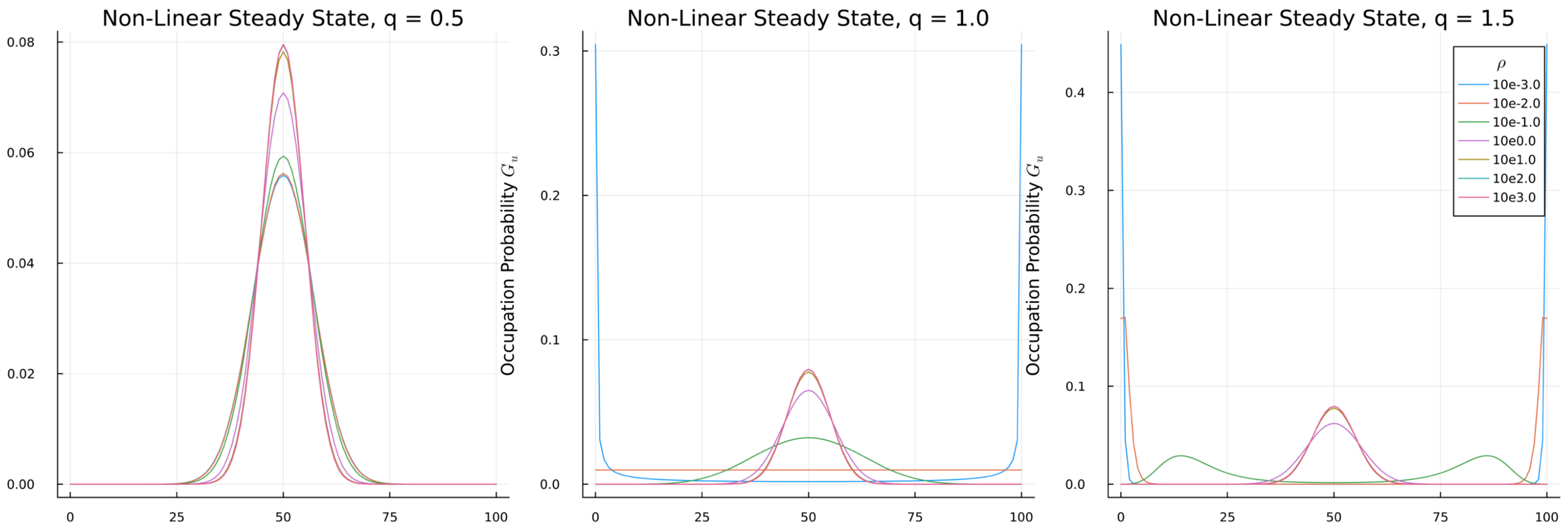
At $\langle k_{ex} \rangle = 0.05$, coexistence only occurs at $n > 50$. At $\langle k_{ex} \rangle = 0.1$, coexistence occurs above $n > 20$



Non Linear Results

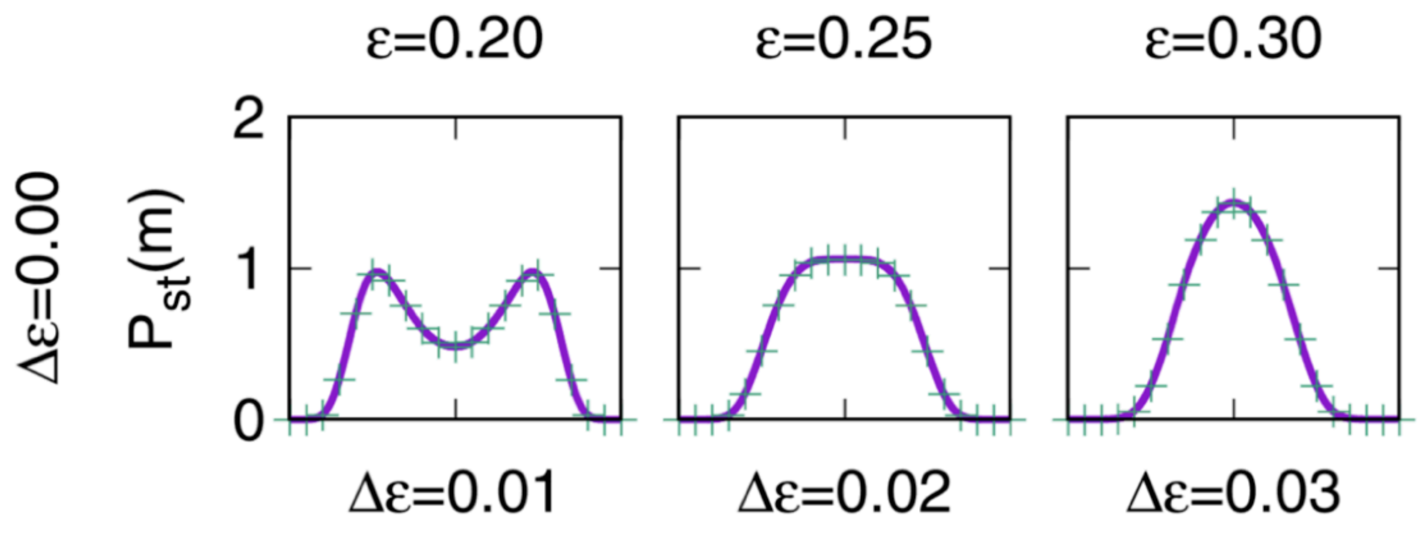
Conformist nodes($q > 1$) create stable minorities

- For $q < 1$ hipster nodes drive model towards coexistence
- For $q > 1$ conformist nodes create a stable minority



Noisy Non Linear Voter Model[a]

1. Give each node a random chance to flip, separate rates for each spin states a_0, a_1.
2. Total noise level $\epsilon \propto a_0 + a_1$
3. Noise asymmetry level $\Delta\epsilon \propto a_0 - a_1$



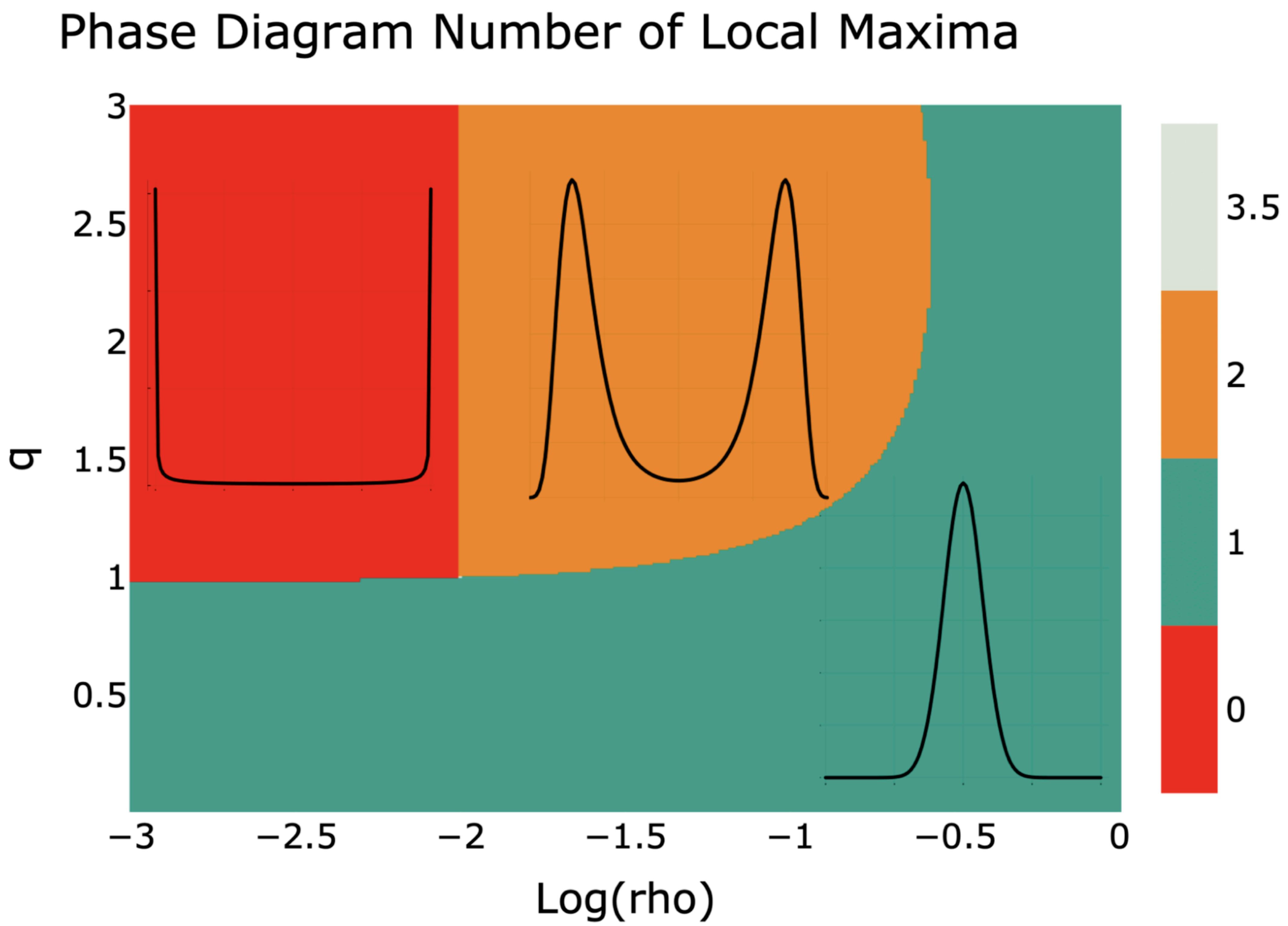
Steady State Distributions

1. Certain noise levels lead to emergence unimodal magnetization distribution
2. Others lead to bimodal distribution

[a] Peralta et al., 2018.

Island of Minority Coexistence

- Let's determine the possible states of the model by plotting the number of local maxima by parameter values
- For low ρ, the critical transition of the original q voter model remains the same.
- At higher couplings $\rho > 0.01$, consensus states become impossible. We see a bimodal distribution where stable minorities coexist within cliques.



Conclusion

Conclusions

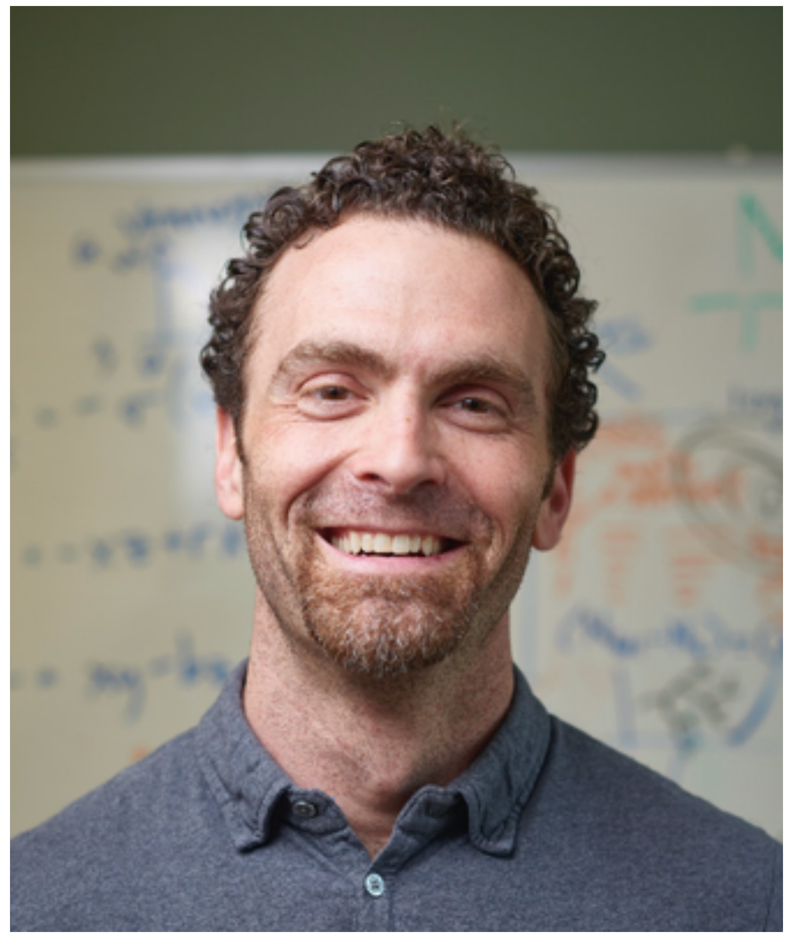
- Formulated dynamics of non-linear voter model on higher order networks using approximate master equations
- Increasing coupling between cliques creates coexistence in steady states
- High q combined with a specific range of couplings allows for stable minority coexistence.

Future Work

- Derive an analytical expression for different phases of model.
- Can the higher order effects be seen as a noise term?
- How does heterogeneity in non-linearity affect results - eg. hipster cliques?
- Compare results with results on pairwise network

Thank You

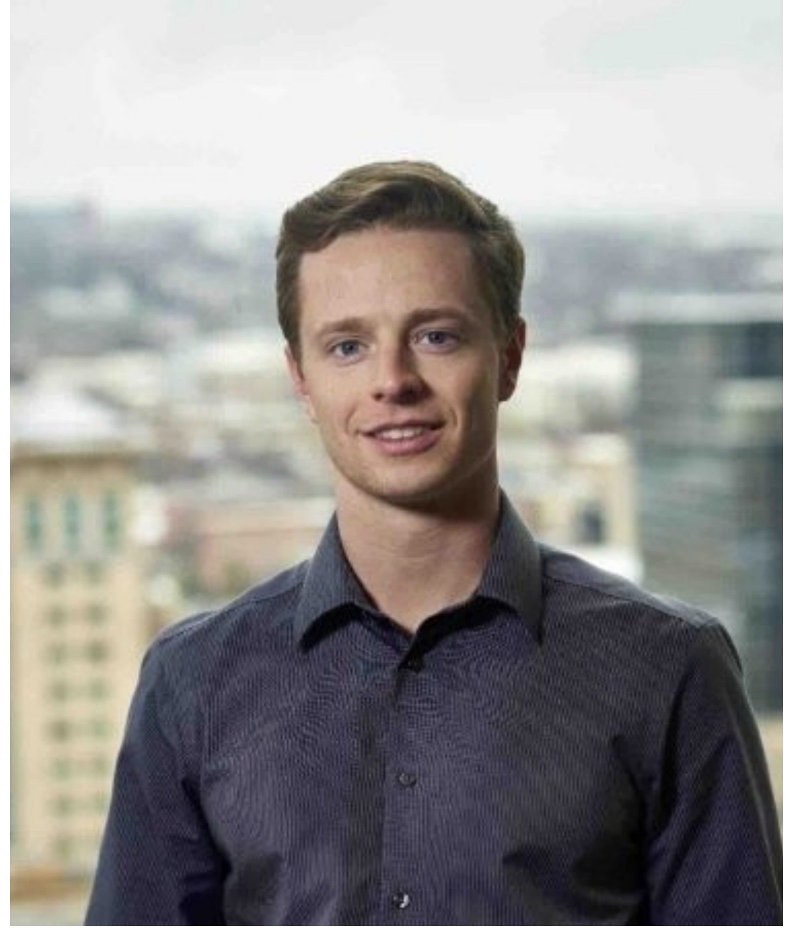
Peter Dodds



Chris Danforth

Laurent Hébert-Dufresne

Nick Cheney



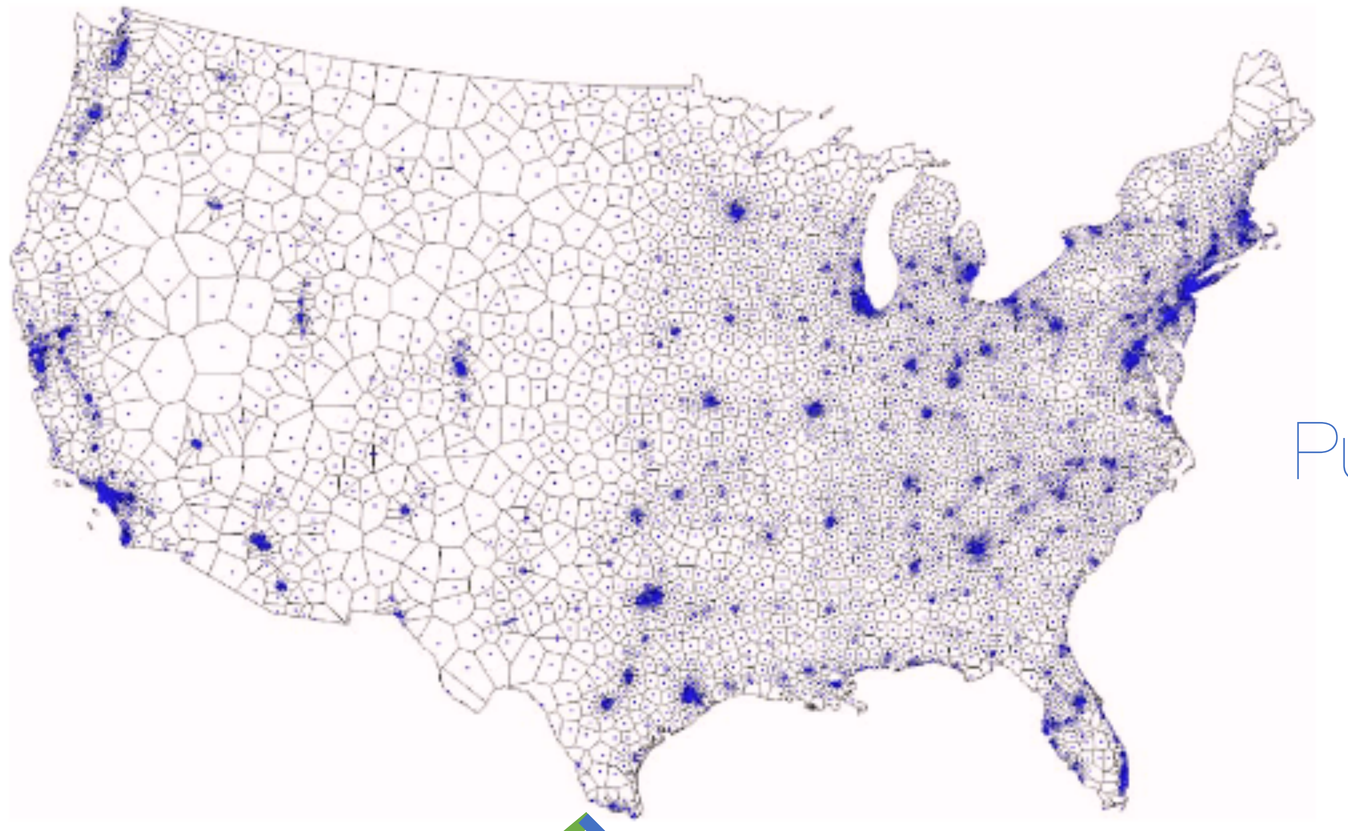
Ethan Ratliff Crain

Alex Friedrichsen

MassMutual

Backup Slides

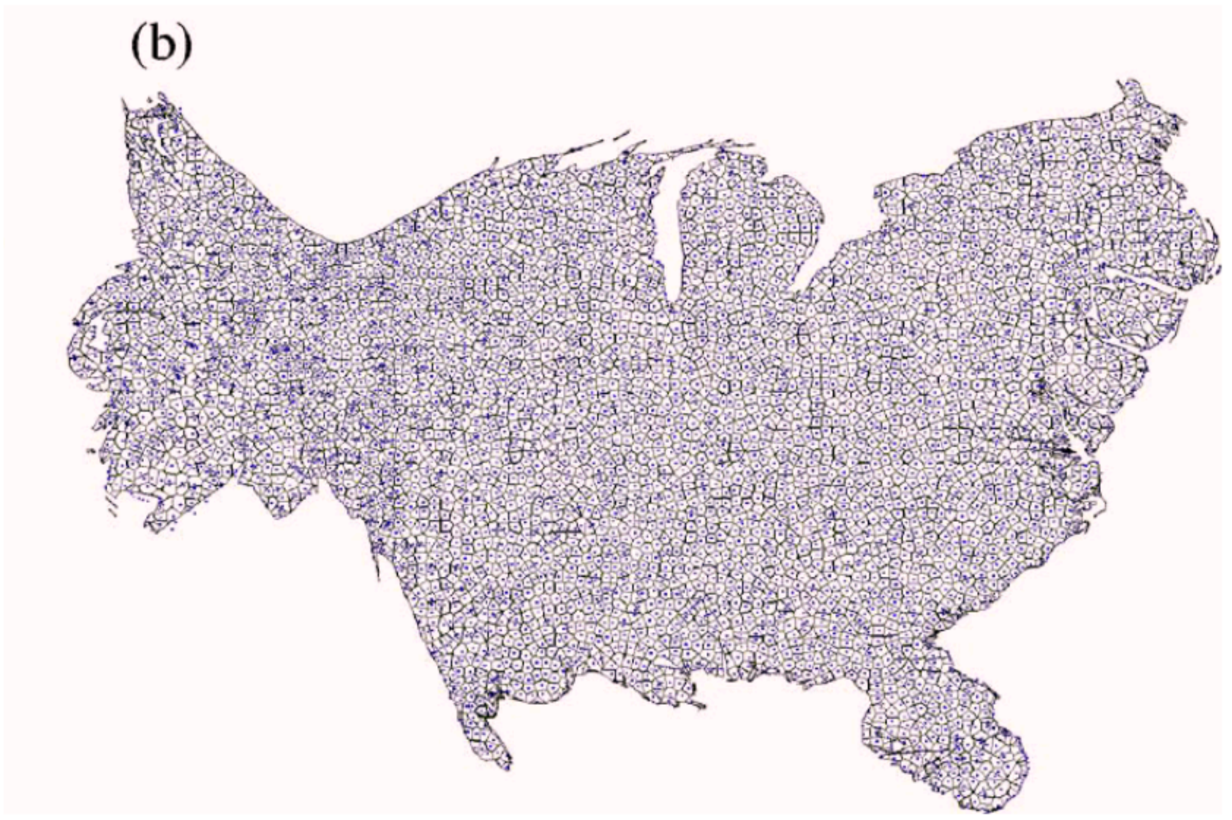
Facility Robustness



Public Facilities with scaling 2/3

Cartogram equalizing ρ

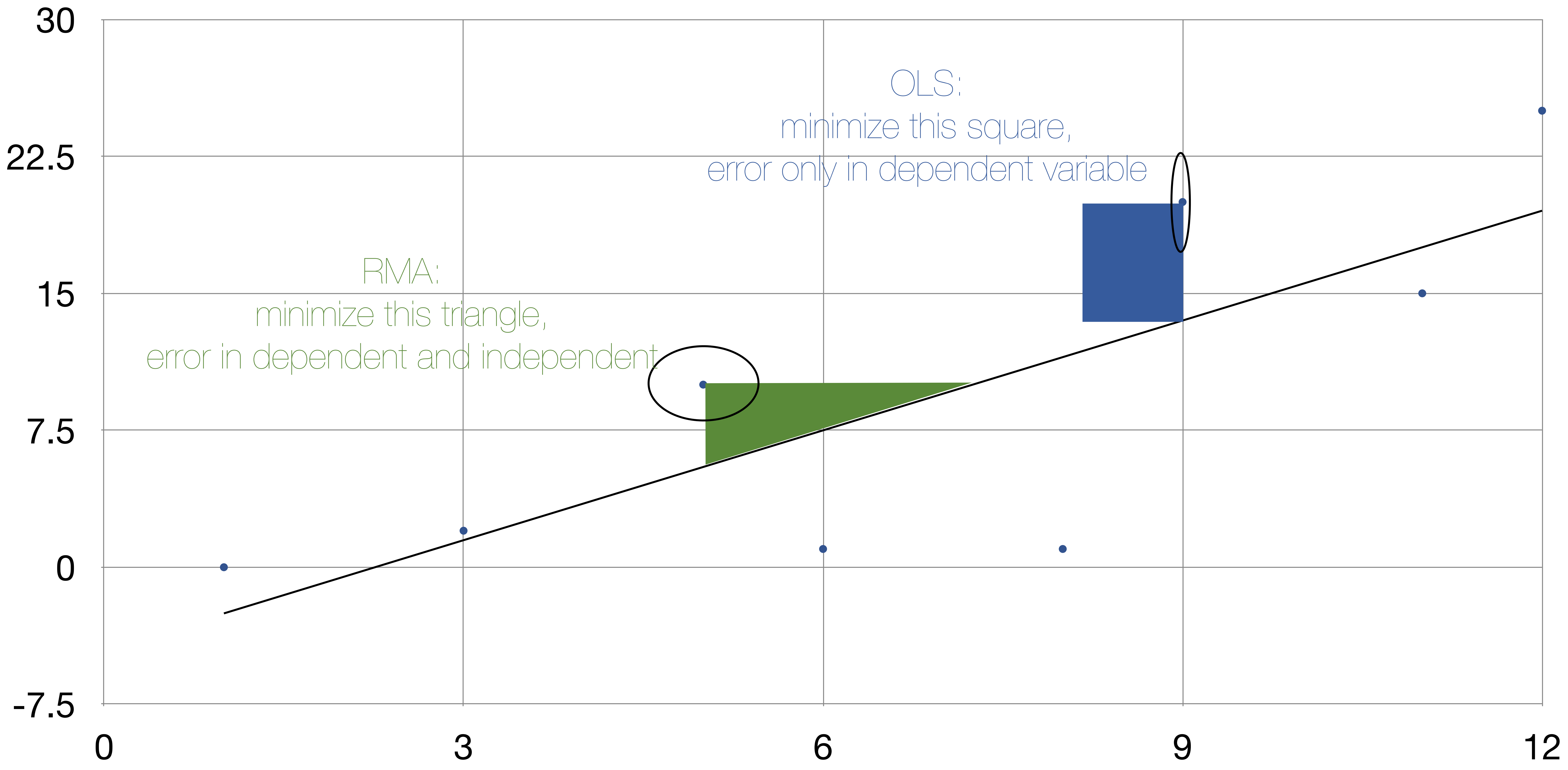
Cartogram equalizing $\rho^{\frac{2}{3}}$



PL

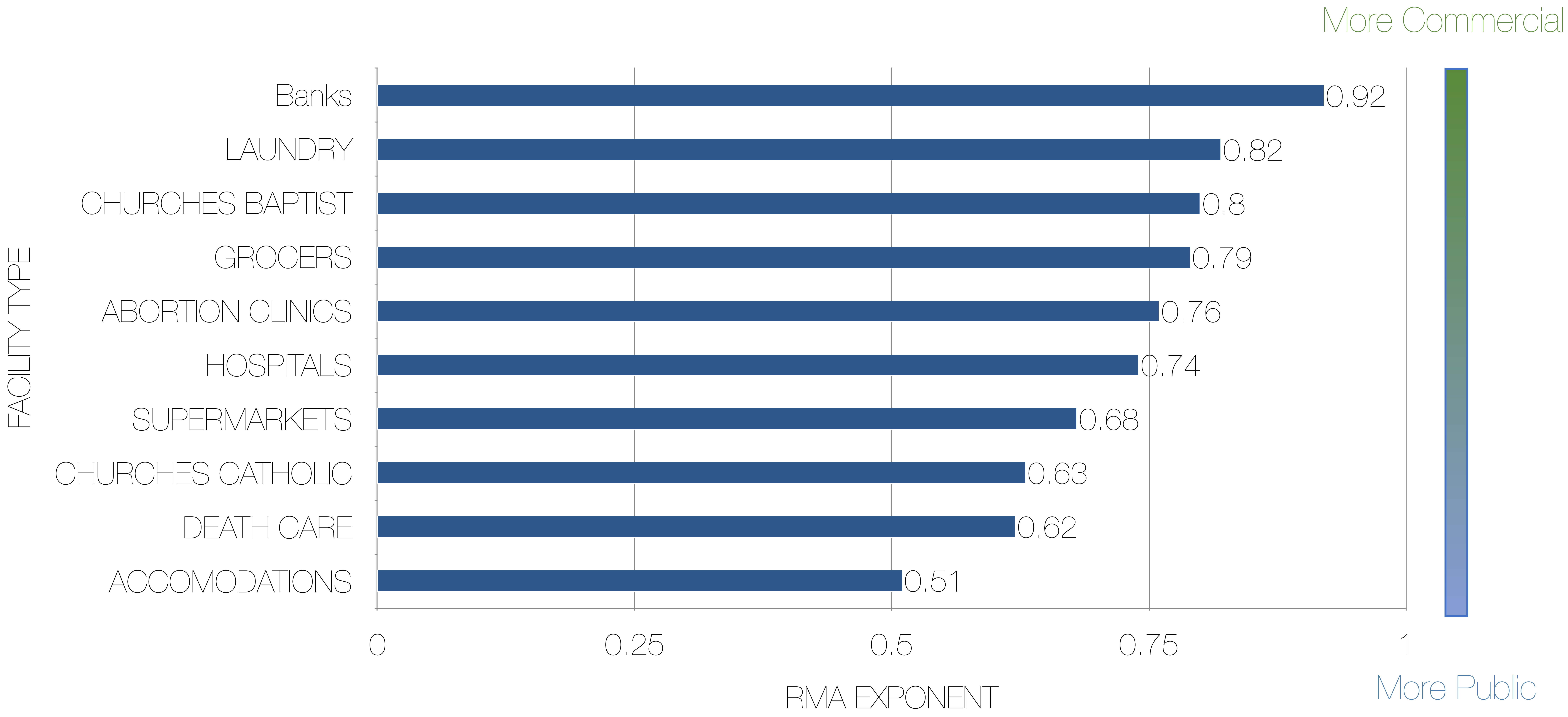
Facilities **partition space equally** when space is **is distorted to equalize the appropriate exponent**

[6]M. T. Gastner and M. E. J. Newman, "Optimal design of spatial distribution networks," Phys. Rev. E, vol. 74, no. 1, p. 016117, Jul. 2006, doi: 10.1103/PhysRevE.74.016117.



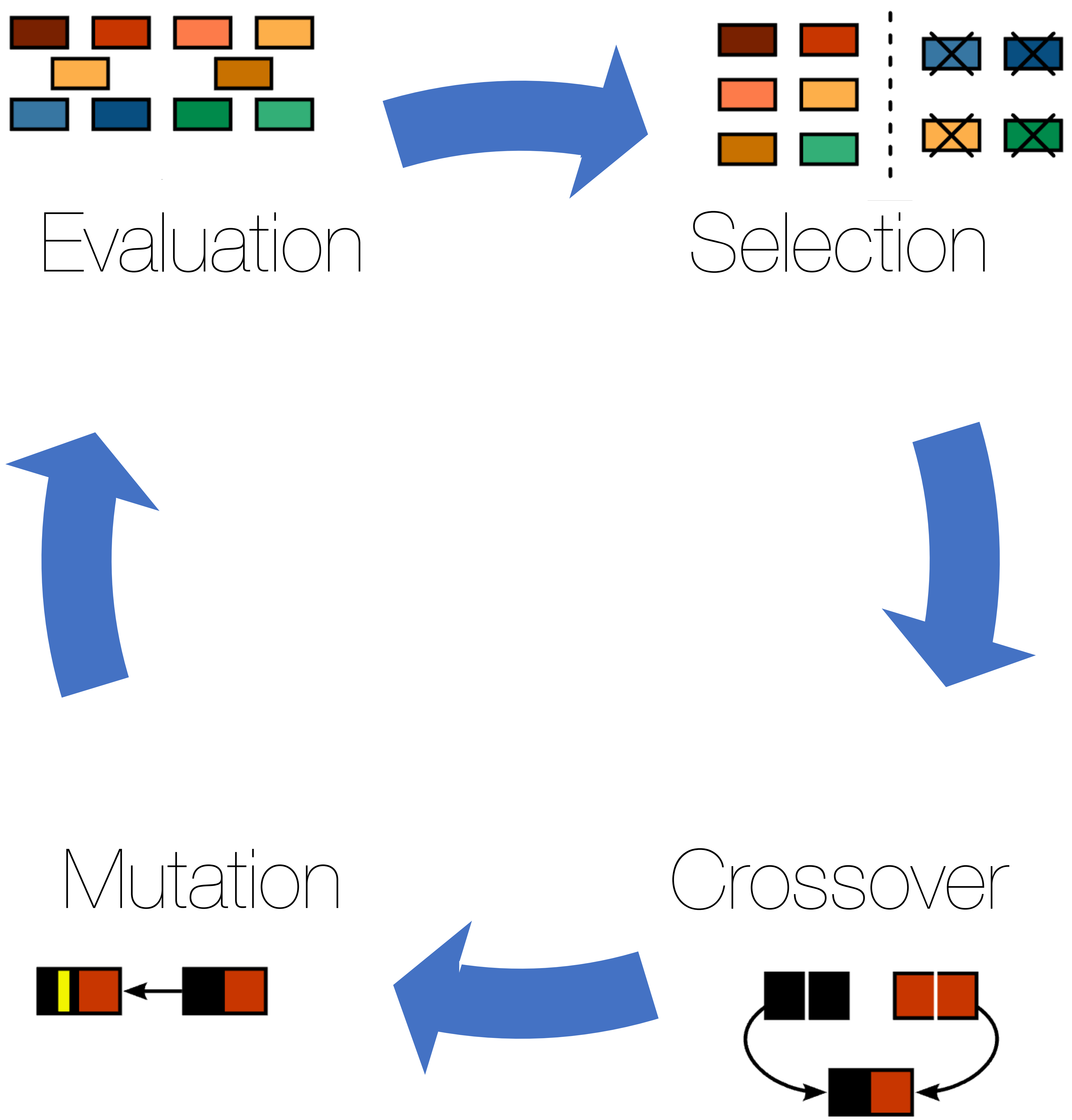
[7] J. M. V. Rayner, "Linear relations in biomechanics: the statistics of scaling functions," Journal of Zoology, vol. 206, no. 3, pp. 415–439, 1985, doi: 10.1111/j.1469-7998.1985.tb05668.x

How do Empirical Facilities Scale?



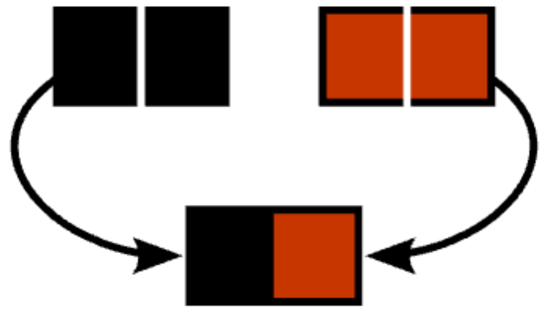
What Are Evolutionary Algorithms?

- Class of optimization algorithms inspired by biological evolution
- Algorithms of last resort
  - only useful when there is no gradient and no information about the fitness landscape
- Solution to a problem are genomes
- Loss function is their fitness
- Solutions are evolved and crossbred to identify better solutions

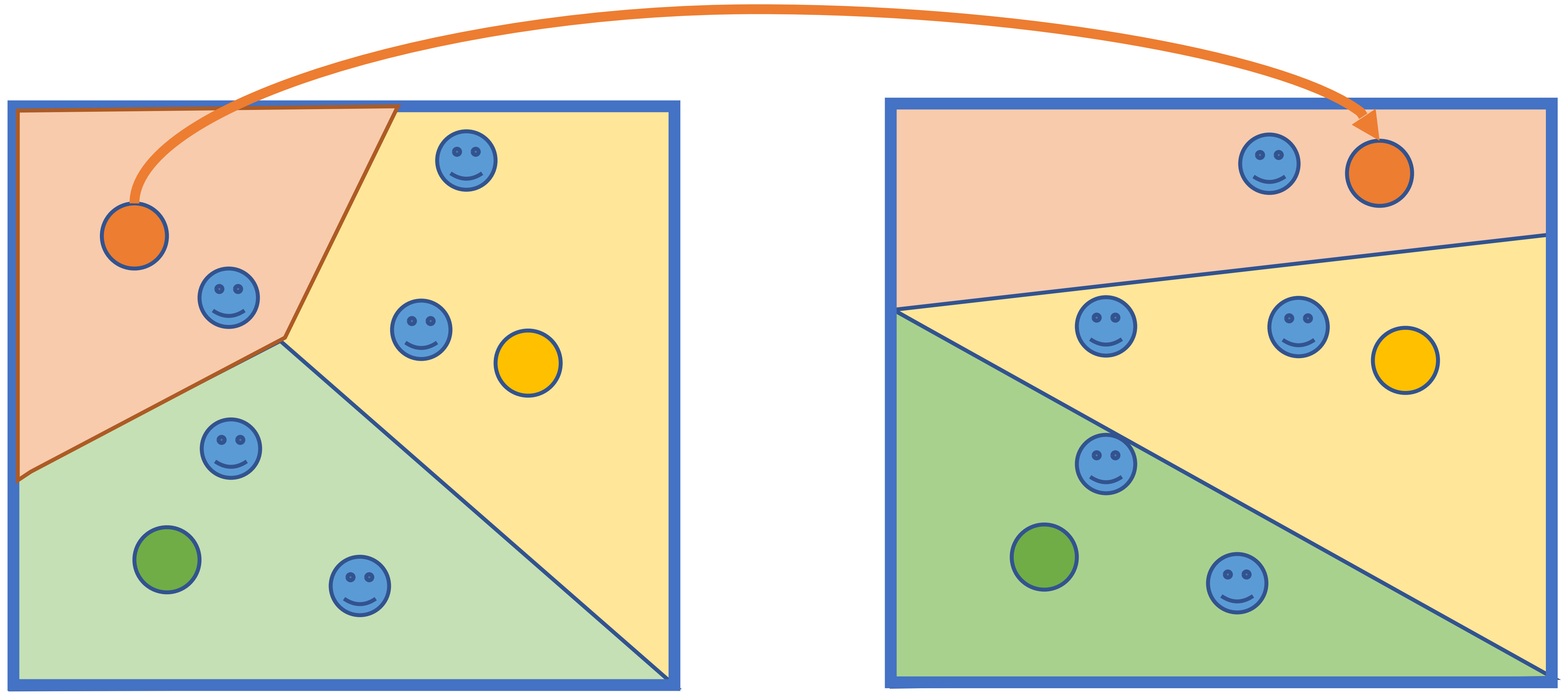


Mutation

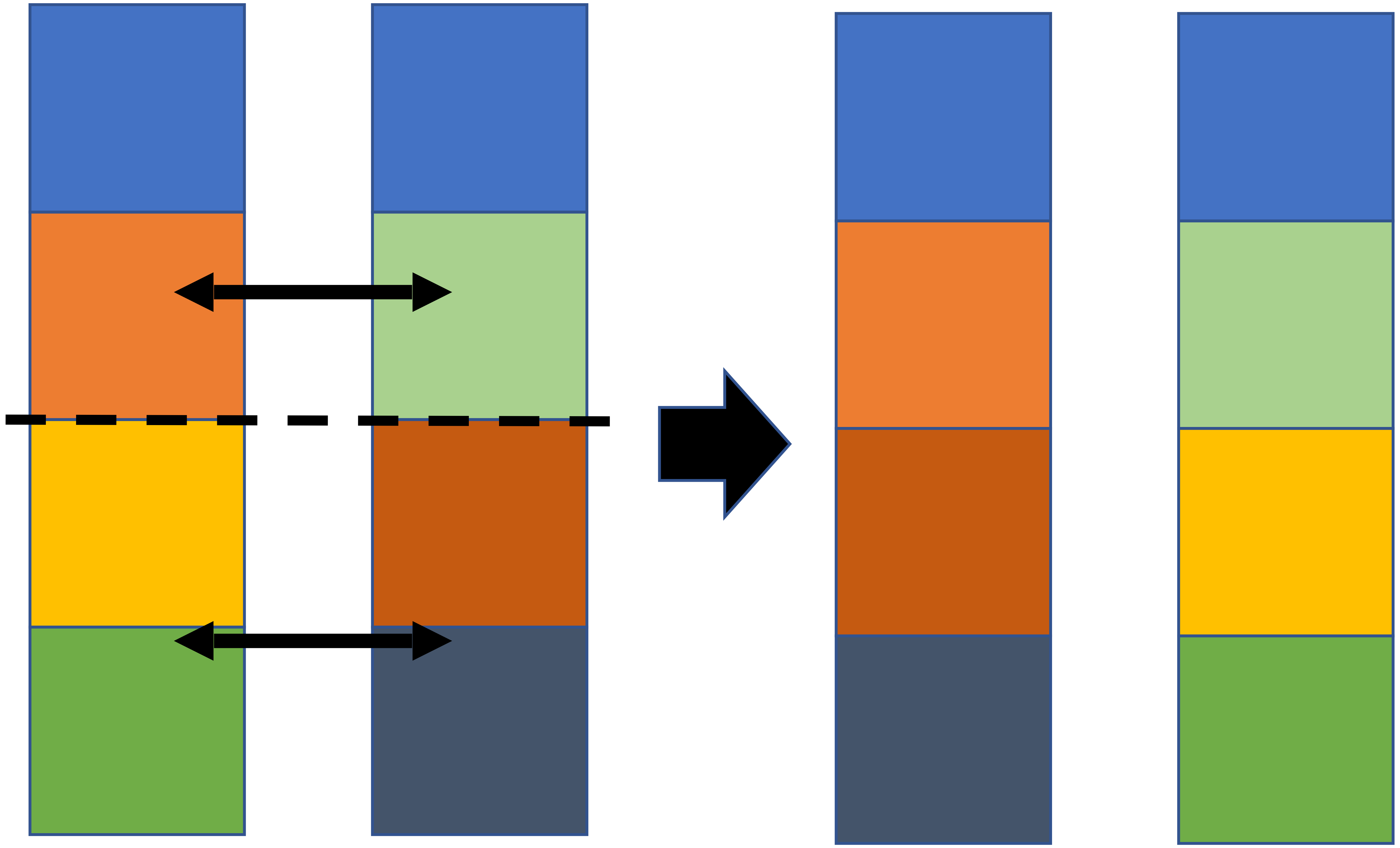
Crossover



Mutation: Randomly relocate a subset of the facilities



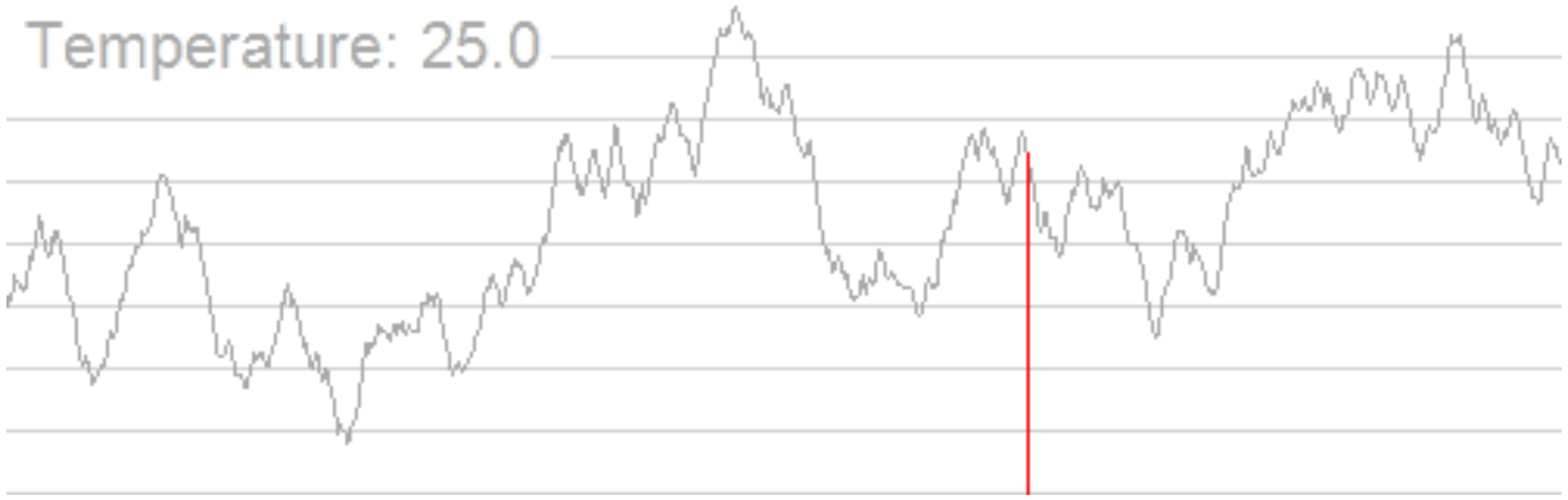
Crossover: Divide the facility at N points, swap



Simulated Annealing

Algorithm 1 Simulated Annealing Without Perturbation

procedure SimulatedAnnealing(Tmax, Tmin, α)
  Initialize solution g
  Evaluate Fitness of solution f $(F(g))$
  $T \leftarrow T_{max}$
  for $i \leftarrow 1$ to *Generation* **do**
    $g' \leftarrow Mutate(g')$
    $\Delta E \leftarrow F(g') - F(g)$
    if $\Delta E < 0$ **then**
      $g \leftarrow g'$
    else
      $p \leftarrow exp(-\Delta E/T)$
      $r \leftarrow random(0, 1)$
      if $r < p$ **then**
        $g \leftarrow g'$
      end if
    end if
    $T \leftarrow \alpha T$
  end for
  return g
end procedure



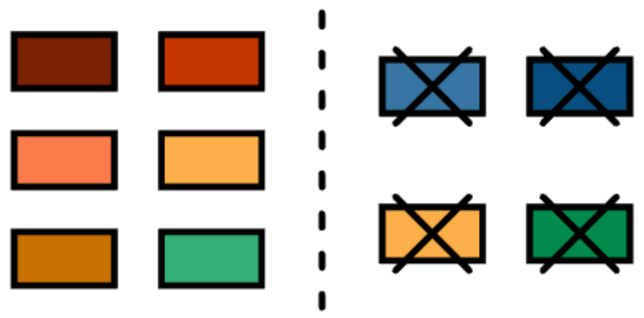
- Randomly mutate a facility
- If the mutated facility has higher fitness, accept it.
- If the mutated facility has lower fitness accept it with a probability $\exp(\frac{F - F'}{T})$
- Where $\mathbf{T}$ is the temperature, decreases by a factor of alpha every generation

$\mu + \lambda$ Evolutionary Strategy

Algorithm 2 $\mu + \lambda$ Evolutionary Strategy Without Perturbation

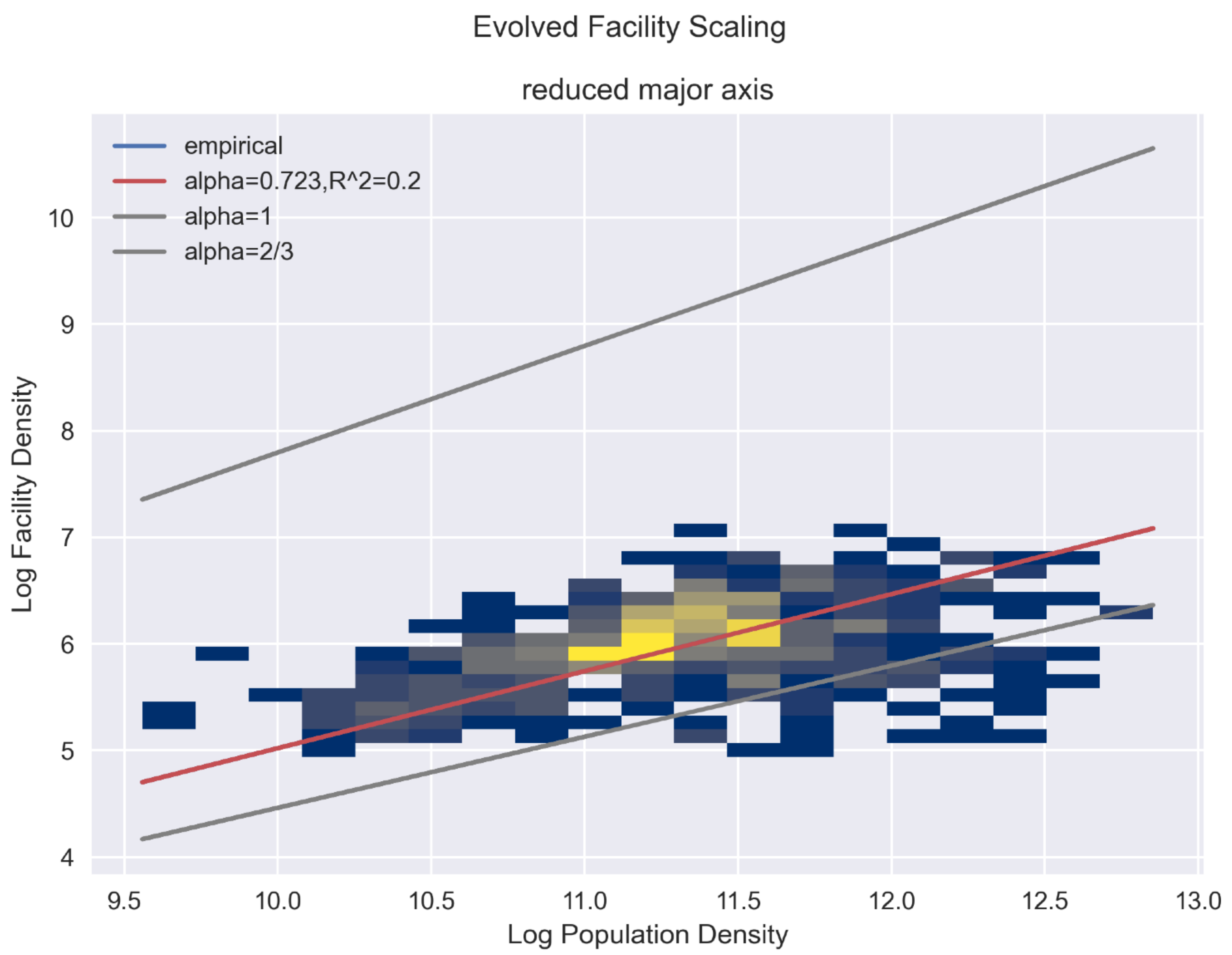
procedure EVOLUTIONARY STRATEGY(mu, lambda)
    $P \leftarrow$ Initialize population of λ individuals
    $Best \leftarrow \square$
    for $i \leftarrow 1$ to $Generation$ **do**
        $Q \leftarrow \{\}$
        for each individual s in S **do**
            $g' \leftarrow Mutate(g)$
            $f_1 \leftarrow$ AssessFitness(g')
            $Q \leftarrow (g', f_1)$ **end for**
            $P \leftarrow P \cup Q$
            $P \leftarrow$ select μ best individuals from P
            $Best \leftarrow$ SelectBestIndividual(P)
        end for
        return $Best$
    end procedure

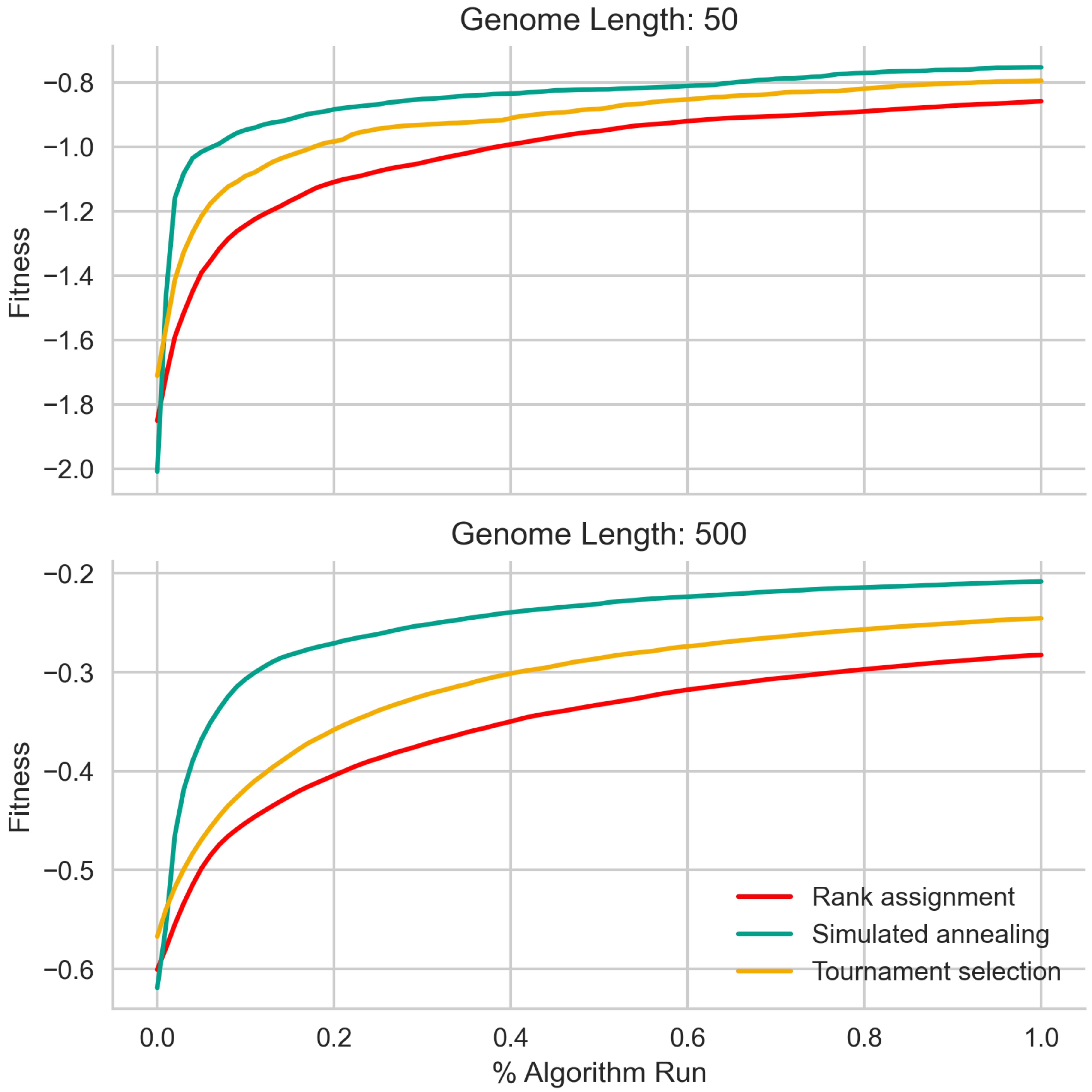
- Randomly mutate all facilities in the population of λ facilities
- Select μ of the best facilities with tournament selection
    - Pairs of head-to-head evaluations with replacement



Q1: **Can evolutionary algorithms identity an ideal facility placement – Yes**

- Simulated annealing outperforms any other metric
- More variation between algorithms – scaling near to optimal
  - Near optimal scaling has ben discovered

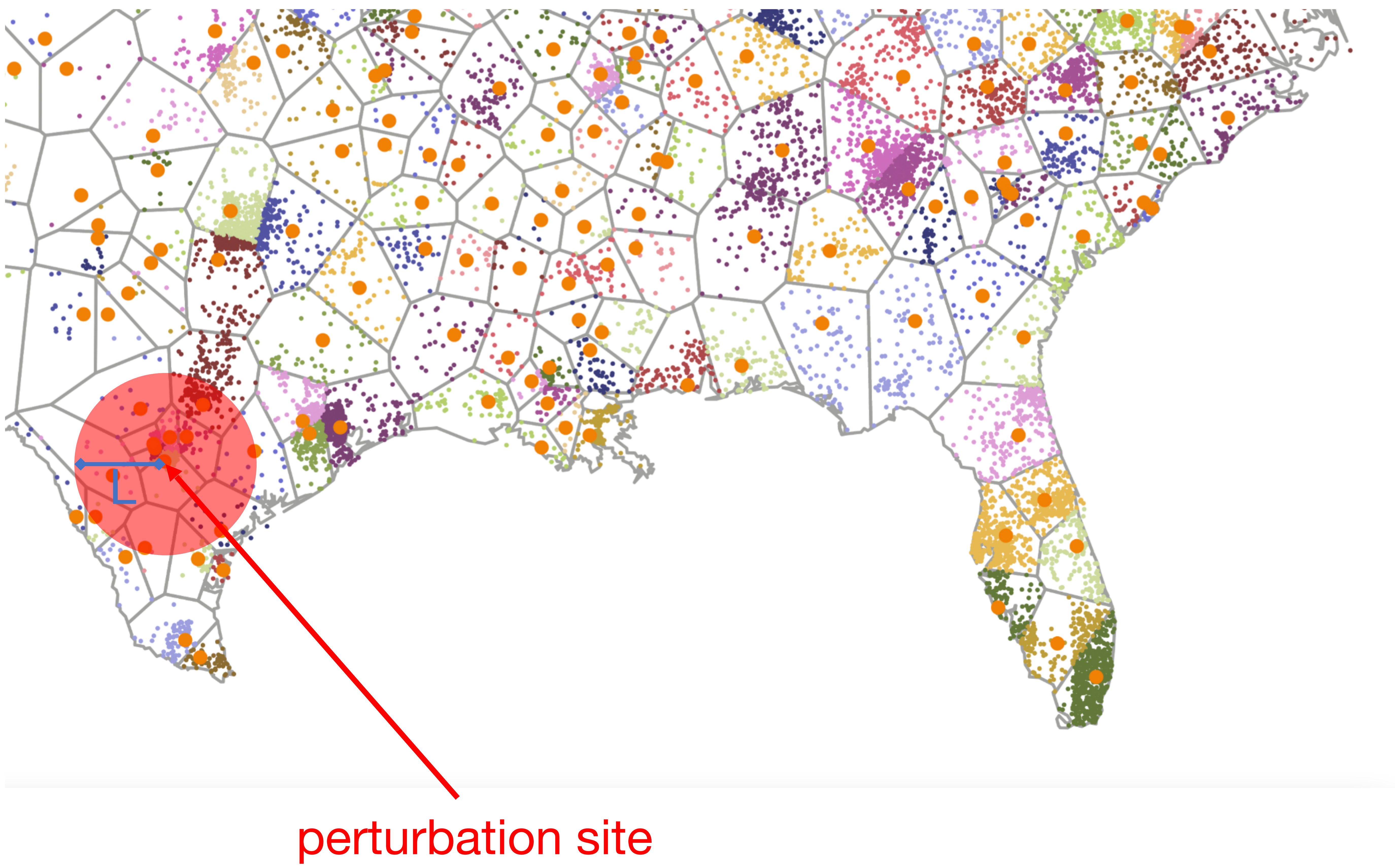




Changes in Supply: Natural Disasters

- **Targeted Removal**
  1. Select a random facility and remove it
- **Radius Based Removal**
  1. Draw a set of catastrophe sites from a specific distribution
  2. Find all the facilities within a distance L of a catastrophe site
  3. Remove them
  4. Calculate the Robustness

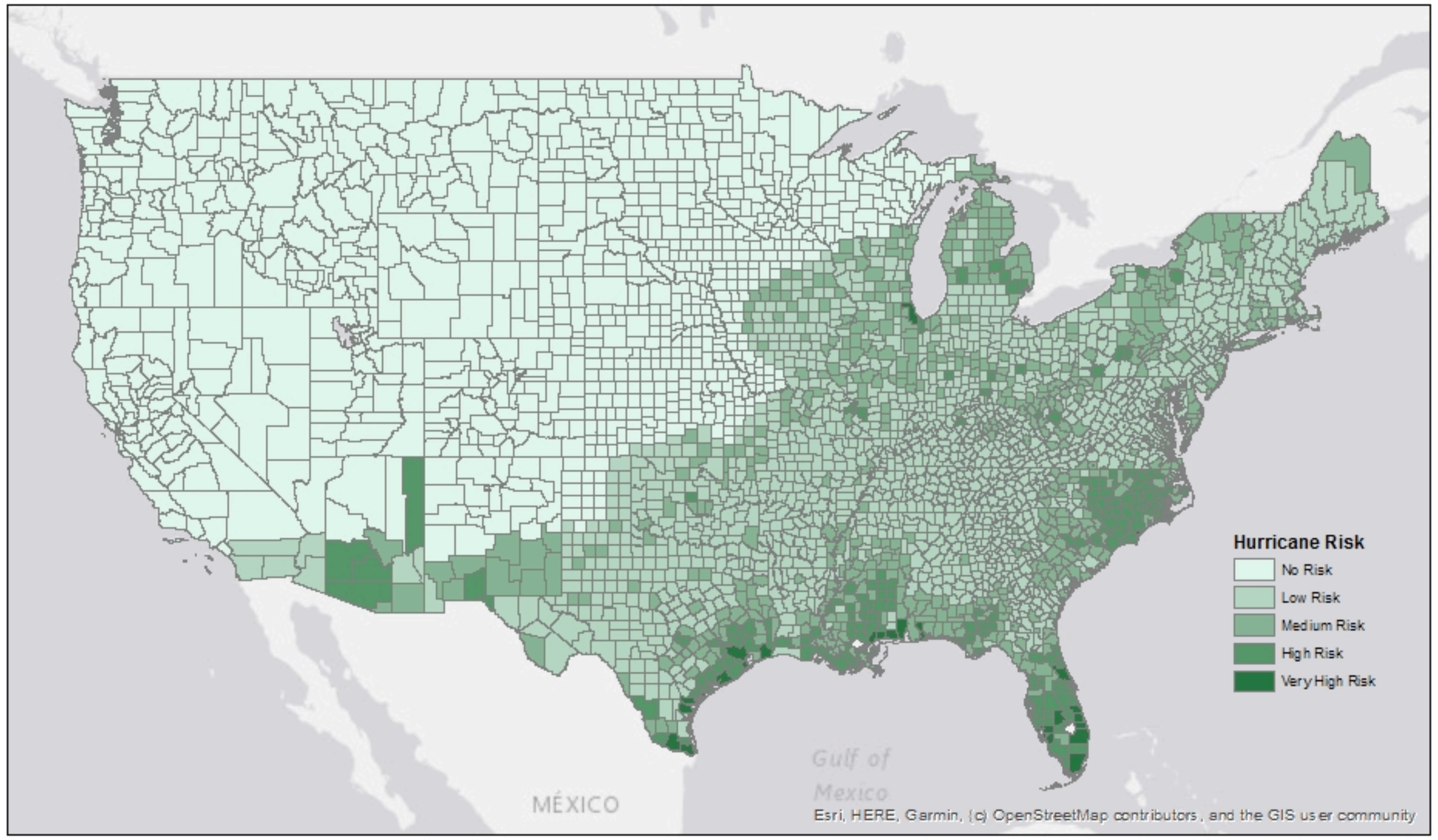
Robustness: How much does the average travel distance increase when facilities are removed?



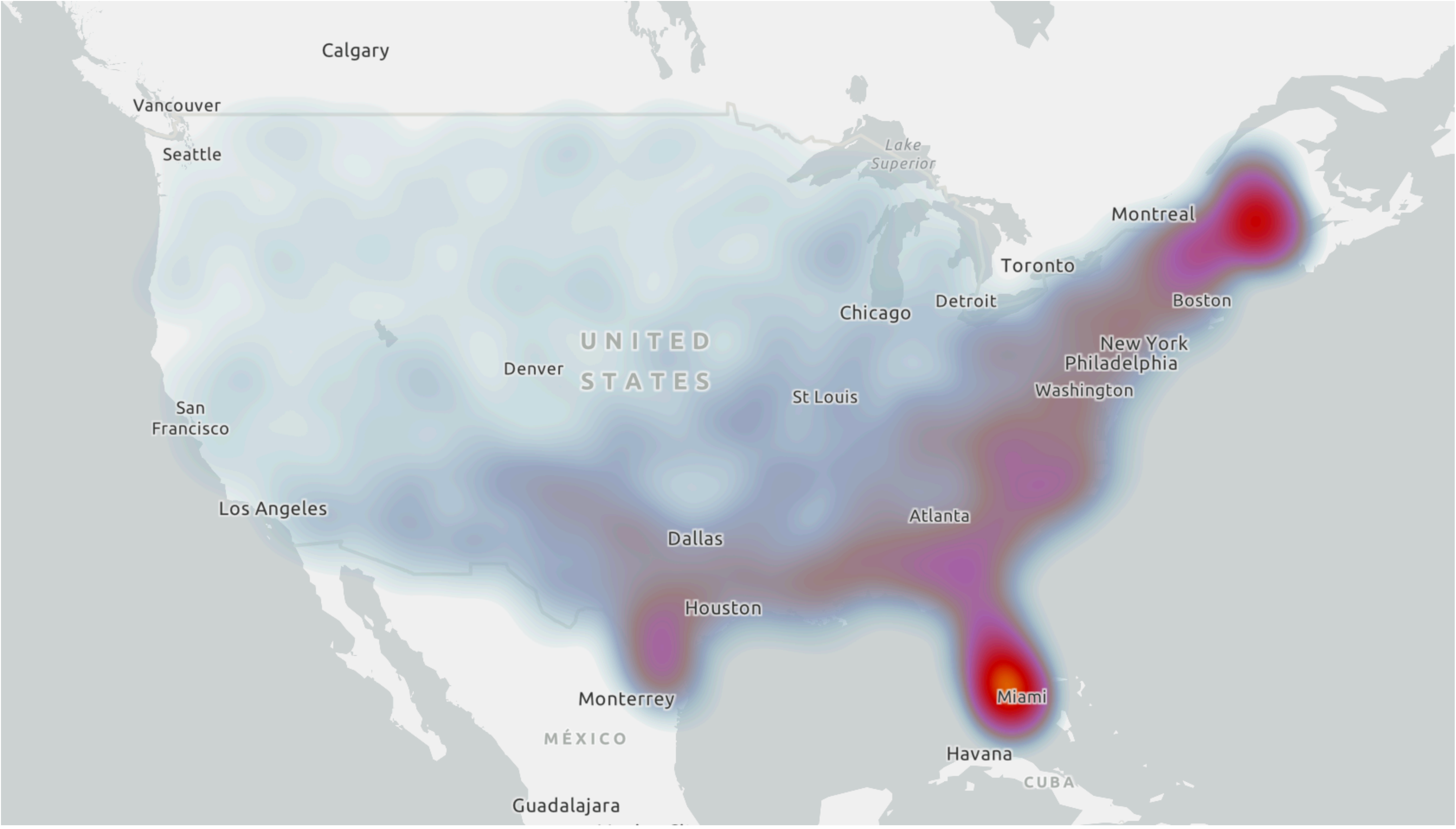
Catastrophe Distribution

- Two choices of $\mathbf{D(r)}$
  - Uniform
  - Pareto Distribution

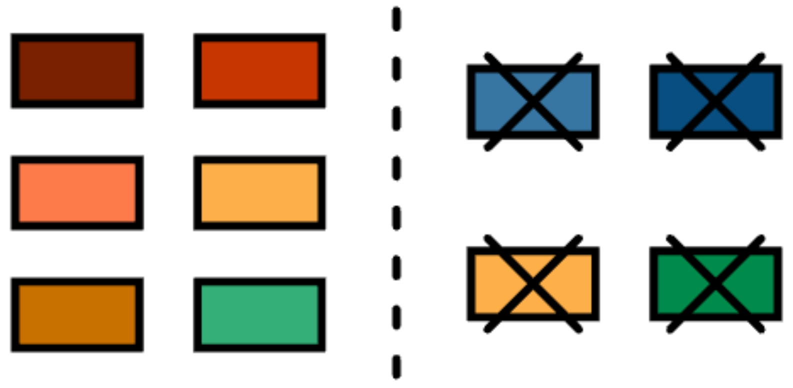
Empirical Flood Distribution

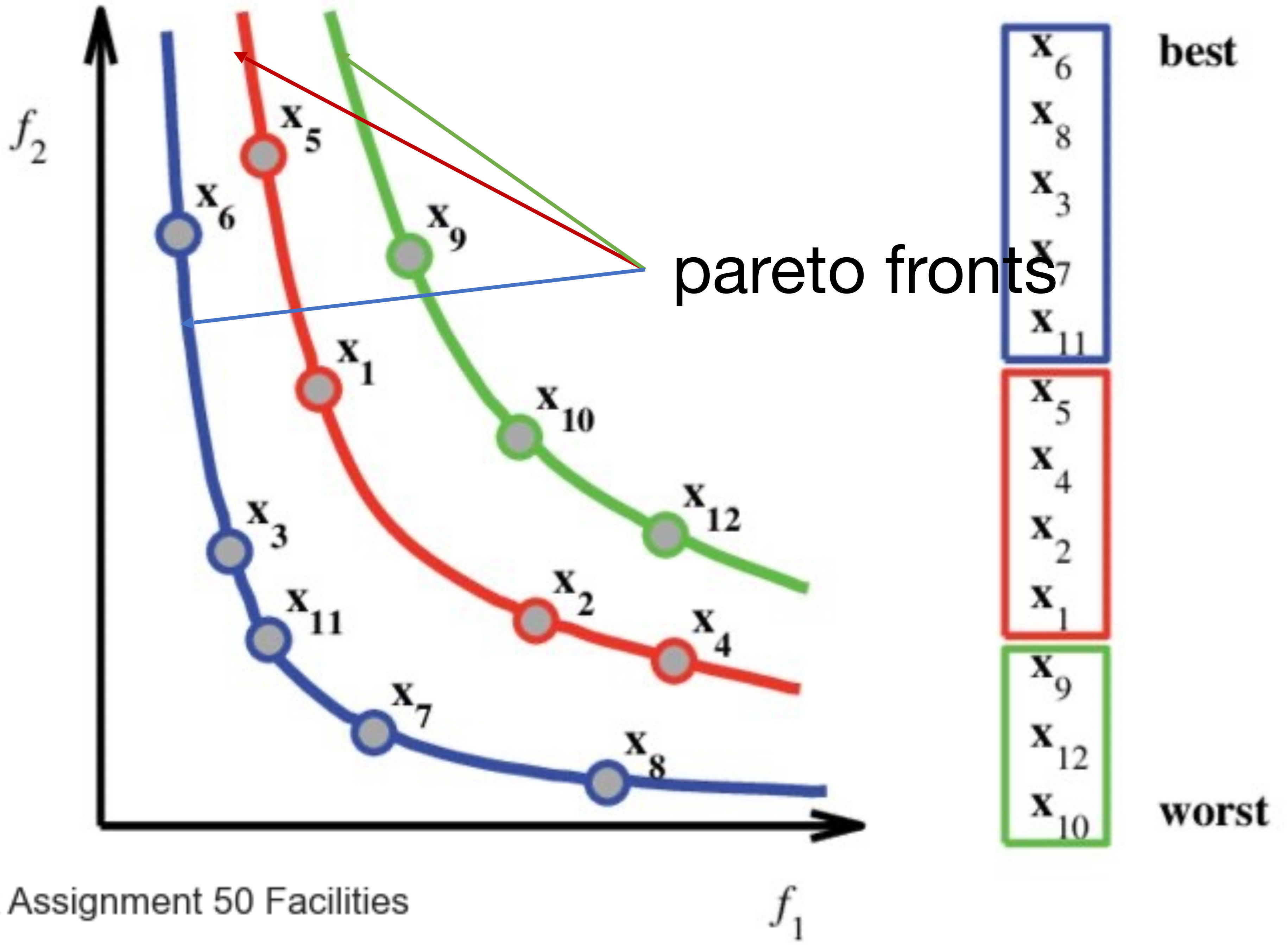


Pareto Distribution



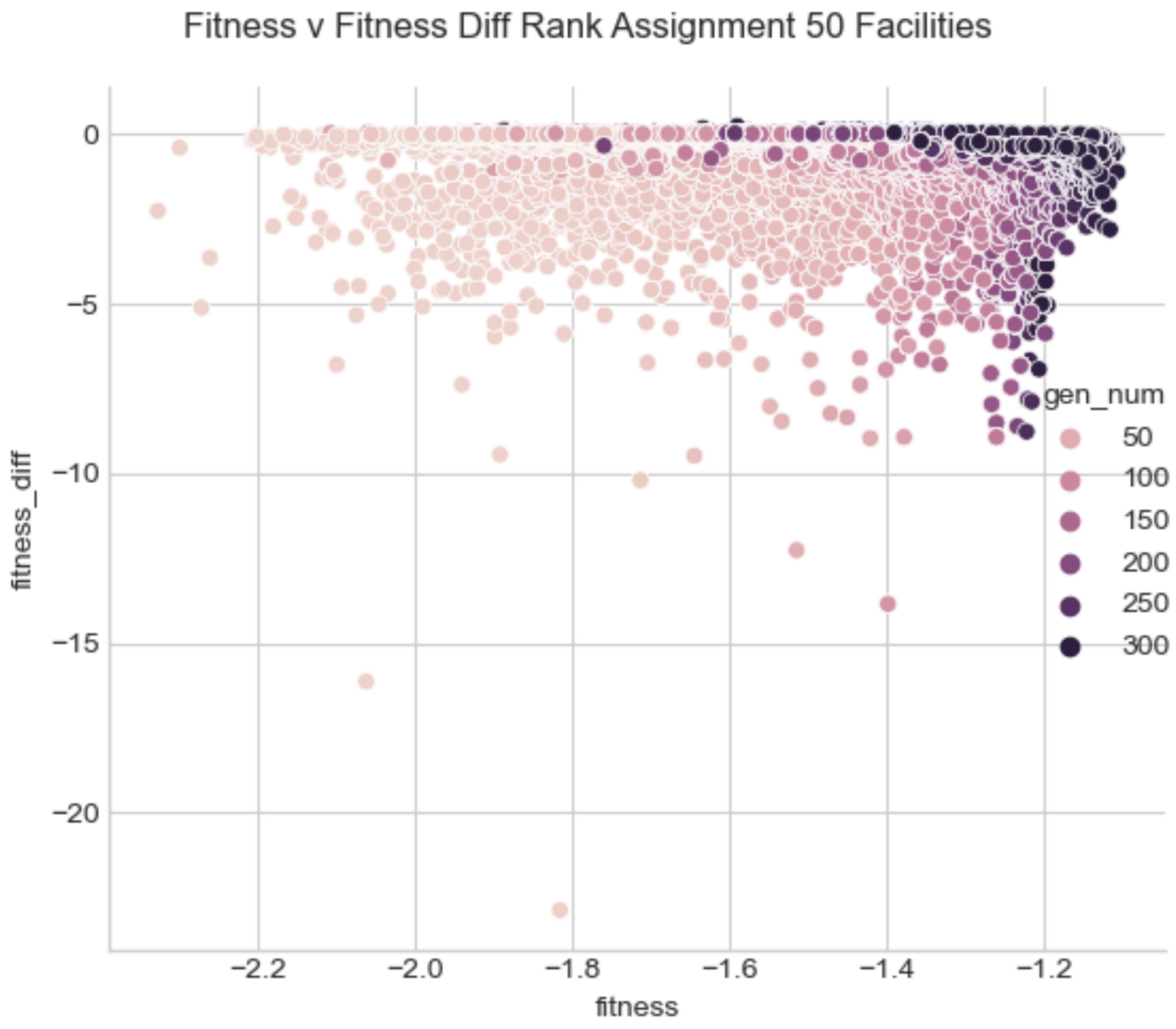
New Selection : Multi-Objective





- Previous selection mechanism: tournament selection on individuals after perturbation
- Multi-objective selection – explicitly incorporate fitness and robustness
- Non-dominated sorting rank assignment
  - Individual i dominates individual j if it is both more fit and more robust
  - Pareto-front of rank k: all individuals not dominated by an individual of a lower rank

- We call $\mu + \lambda$ algorithm with single objective tournament selection – tournament selection
- We call $\mu + \lambda$ algorithm with multi-objective rank assignment – rank assignment



How do we add perturbation to the algorithm?

Algorithm 3 Simulated Annealing With Perturbation

procedure SimulatedAnnealing(Tmax, Tmin, α)
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  Evaluate Fitness of solution f ($F(g)$)
  $T \leftarrow T_{max}$
  for $i \leftarrow 1$ to *Generation* **do**
    $g' \leftarrow Mutate(g')$
    $g'' \leftarrow Perturb(g')$
    $\Delta E \leftarrow F(g'') - F(g)$
    if $\Delta E < 0$ **then**
      $g \leftarrow g'$
    else
      $p \leftarrow exp(-\Delta E/T)$
      $r \leftarrow random(0, 1)$
      if $r < p$ **then**
        $g \leftarrow g'$
      end if
    end if
    $T \leftarrow \alpha T$
  end for
  return g
end procedure

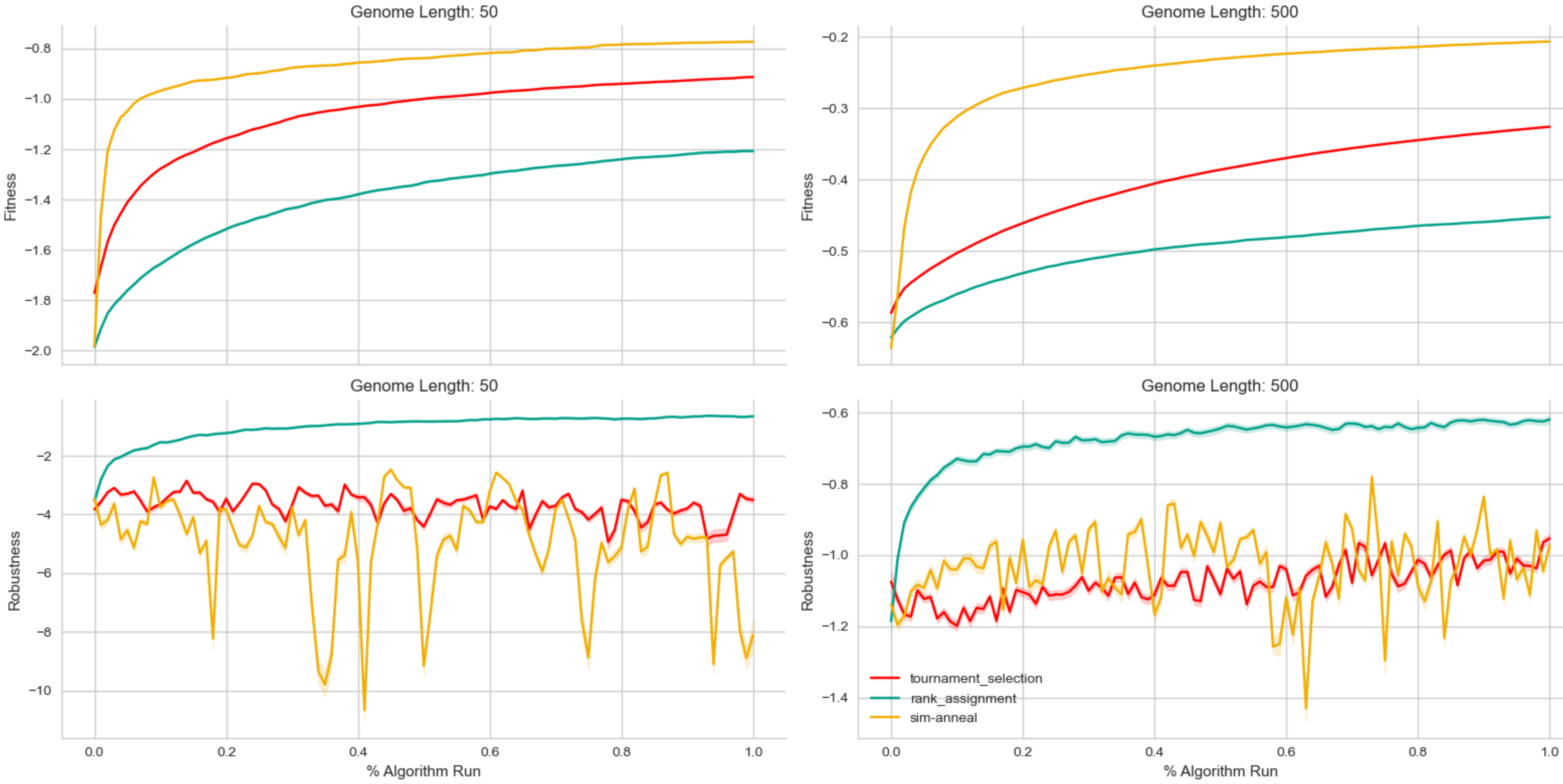
Assess fitness of individual after perturbation - Implicitly select for robust solutions

Algorithm 4 $\mu + \lambda$ Evolutionary Strategy With Perturbation

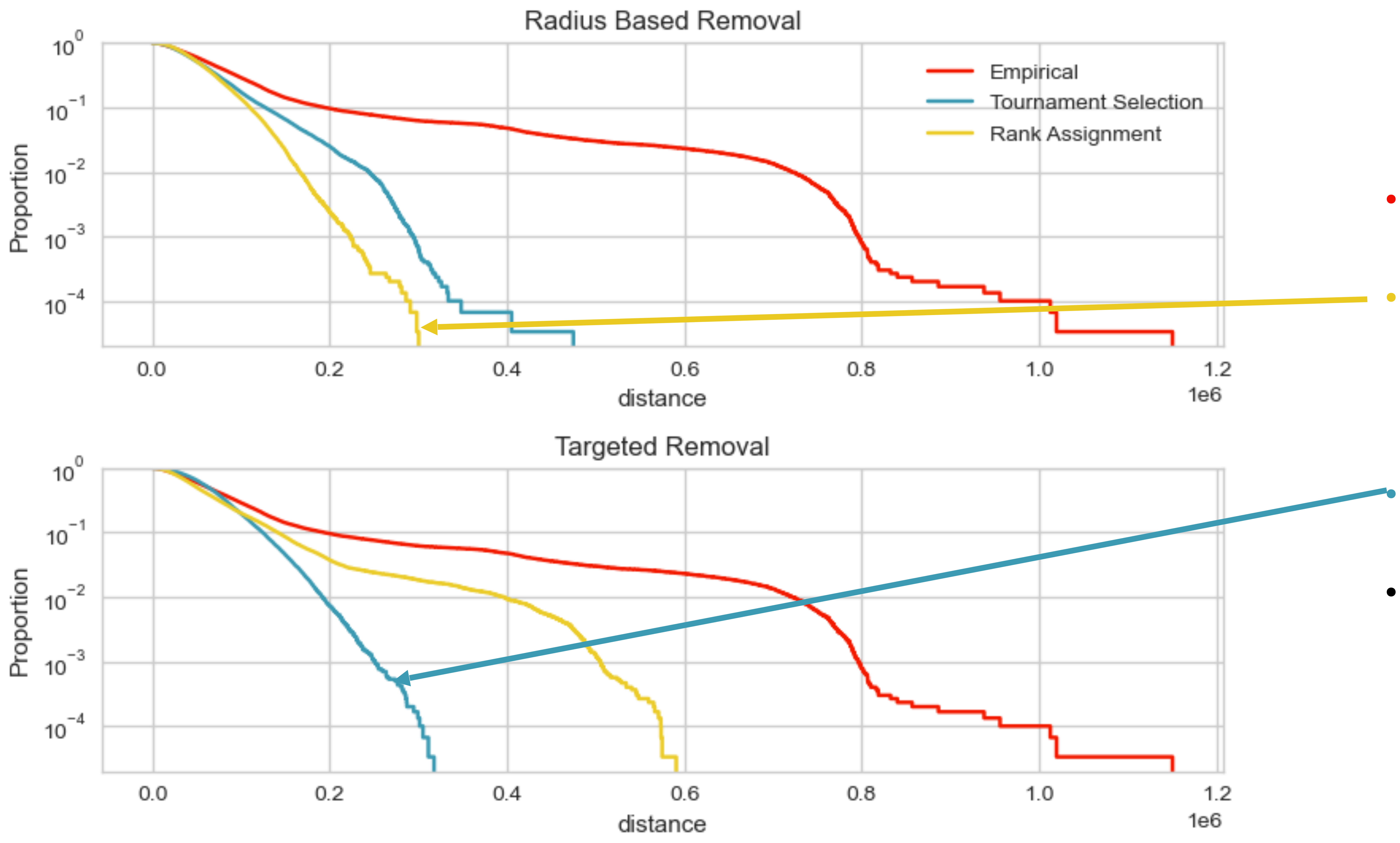
procedure Evolutionary Strategy(mu, lambda)
  $P \leftarrow$ Initialize population of λ individuals
  $Best \leftarrow \square$
  for $i \leftarrow 1$ to *Generation* **do**
    $Q \leftarrow \{\}$
    for each individual s in S **do**
      $g' \leftarrow Mutate(g)$
      $f_1 \leftarrow \text{AssessFitness}(g')$
      $g'' \leftarrow \text{Perturb}(g')$
      $f_2 \leftarrow \text{AssessFitness}(g'')$
      $R \leftarrow f_2 - f_1$ ▷ calculate the robustness
      $Q \leftarrow (g'', f_1, R)$
    end for
    $P \leftarrow P \cup Q$
    $P \leftarrow$ select μ best individuals from P
    $Best \leftarrow$ SelectBestIndividual(P)
  end for
  return *Best*
end procedure

Q2: Can we evolve a robust layout of facilities – Yes!

- Simulated annealing perform well achieves highest absolute fitness
- But simulated annealing and tournament selection fail to achieve increase in robustness
    - Only rank assignment increase robustness



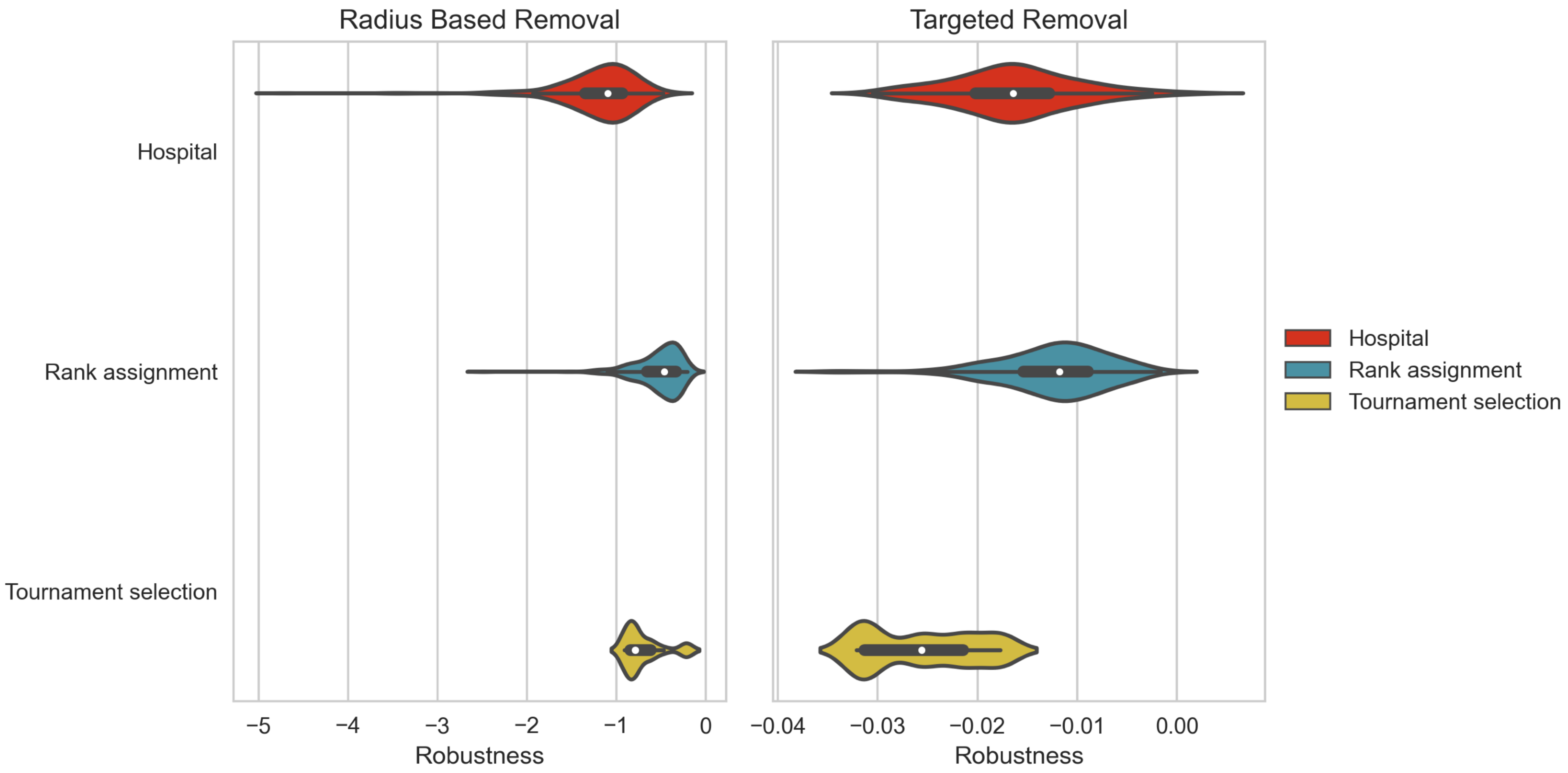
Travel Distance Comparison



- Empirical distribution is much more heavy tailed
- rank assignment outperforms tournament selection for for radius based removal
- Vice-versa for targeted removal
- Is targeted removal harder than radius based

Robustness Comparison

- Rank assignment more robust than empirical layout
- Evolved distributions are much less heavy tailed
- Tournament selection performs poorly



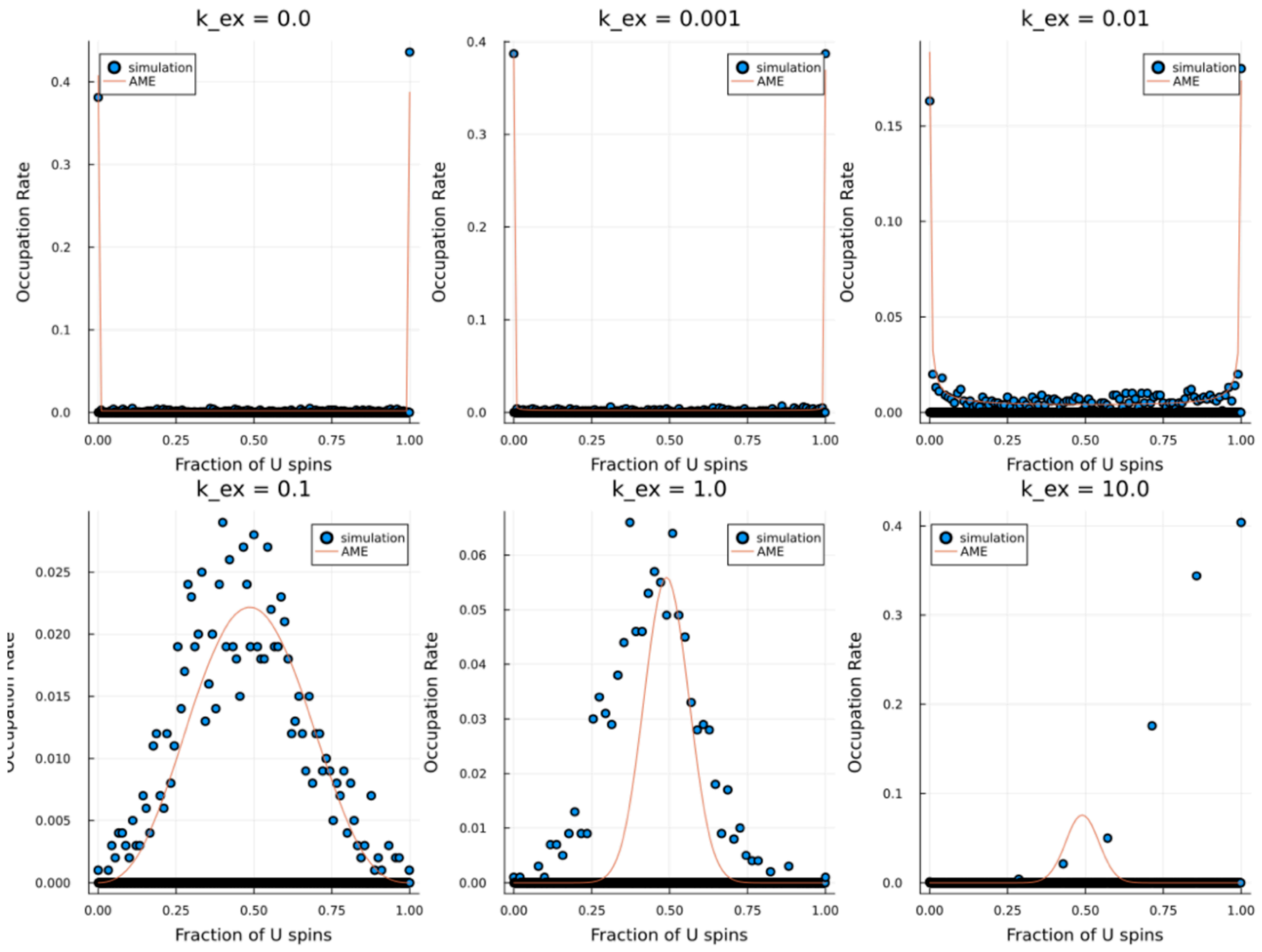
Voter Model

Conclusion

Numerical and Simulation

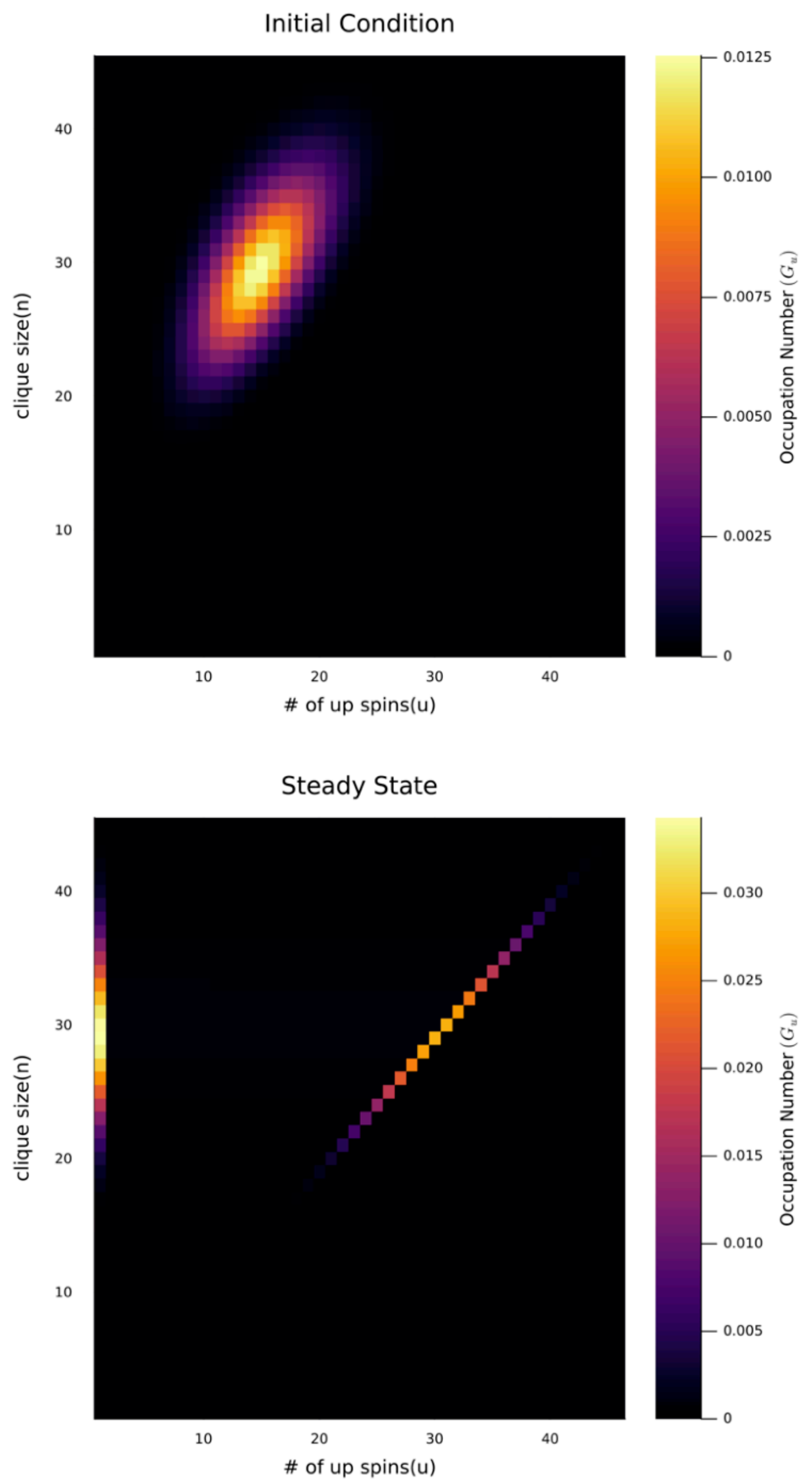
Validating with Simulation

- AME distributions and simulation match for small couplings .
- Simulation and AME diverge for large couplings ($\langle k_{ex} \rangle = 10.0$)
- Discrepancy is possible due to finite size effects



Groups across scale

- Let's relax the assumption of fixed clique size. How do heterogenous groups interacting across scales affect the steady-state dynamics and the possibility of coexistence?
- $G_{u,n}$: the occupation number for groups of size n with u up spins.



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