



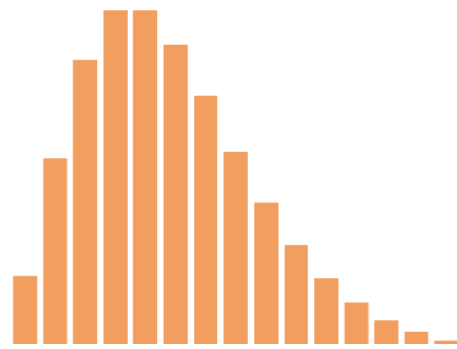
Sensitivity of Epidemic Forecasts with Probability Generating Functions

Mariah C. Boudreau¹, **Will H. W. Thompson¹**, Christopher M. Danforth¹, Jean Gabriel Young¹, Laurent Hébert-Dufresne¹

1: 1. Vermont Complex Systems Institute at University of Vermont, Burlington Vermont

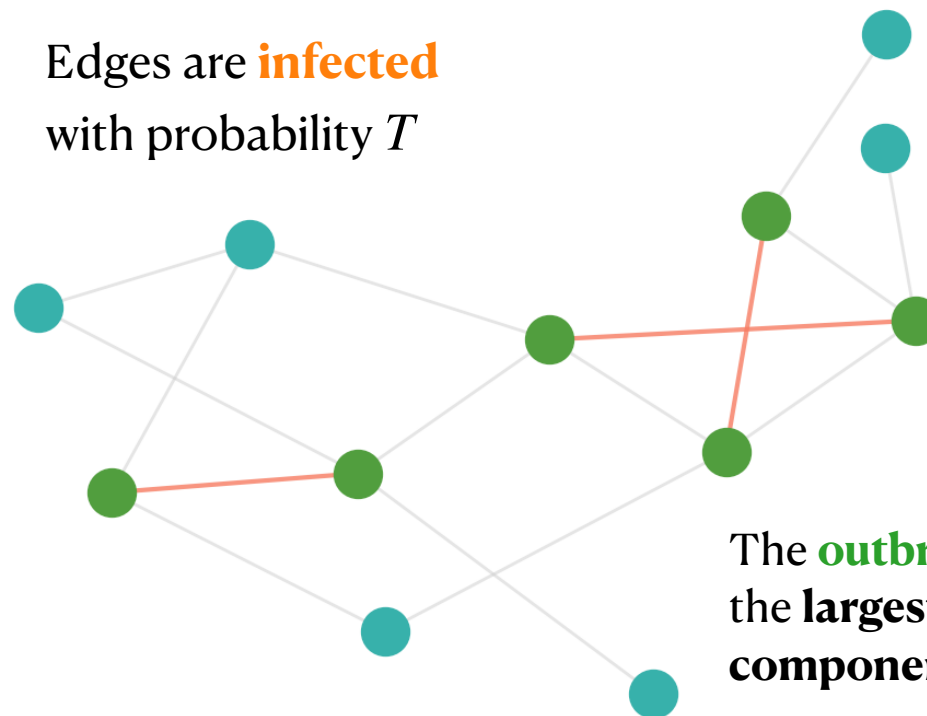
Question:
How **sensitive** are estimates of contagion **outbreak size** to **measurement error**?

Contagion as **percolation**



Node degree sampled from
a **probability distribution**

Edges are **infected**
with probability T



The **outbreak** is the size of
the **largest connected component**

We represent the **probability distribution** with a **probability generating function(PGF)**

$$G_0(x) = p_1x^1 + p_2x^2 + p_3x^3 \dots p_kx^k$$

p_k : probability a
randomly chosen node
has k neighbors

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Branching Process :

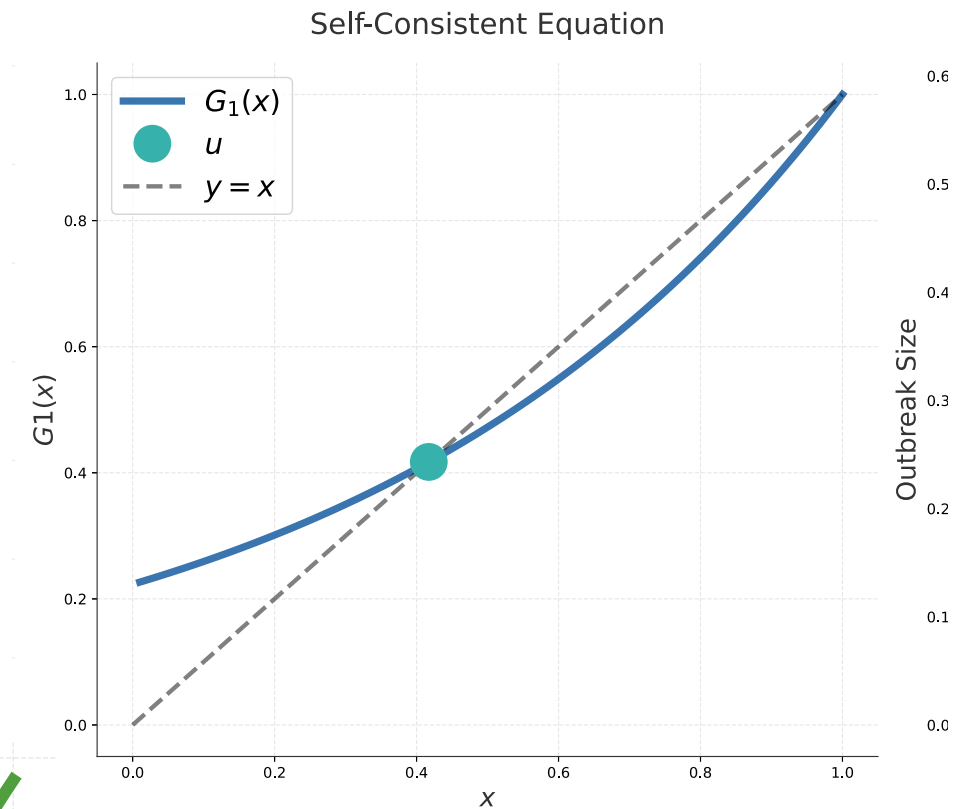
p_k : probability of k infections in next generation

Percolation :

p_k : probability a randomly chosen node has k neighbors

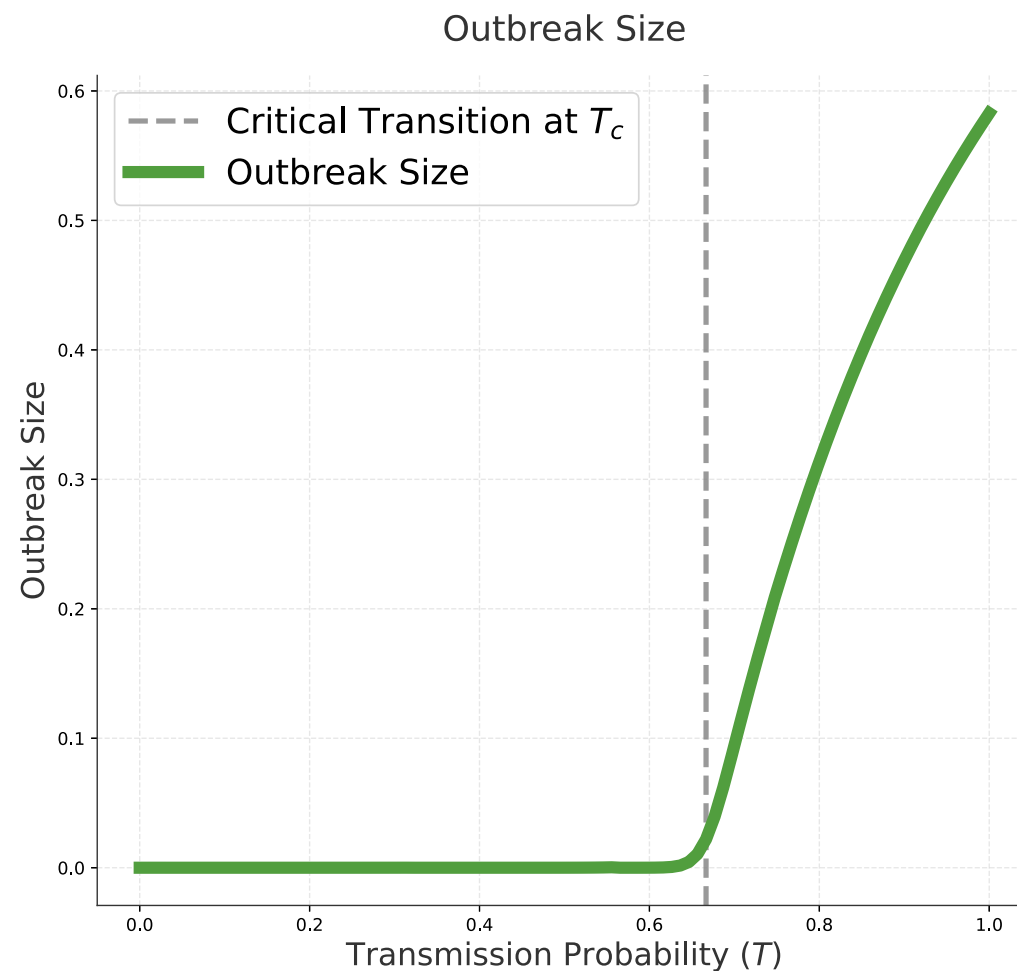
The **average size of an outbreak** is related to the solution to a **self consistent equation**

$$u = G_1(u)$$



$$\langle s \rangle = 1 - u$$

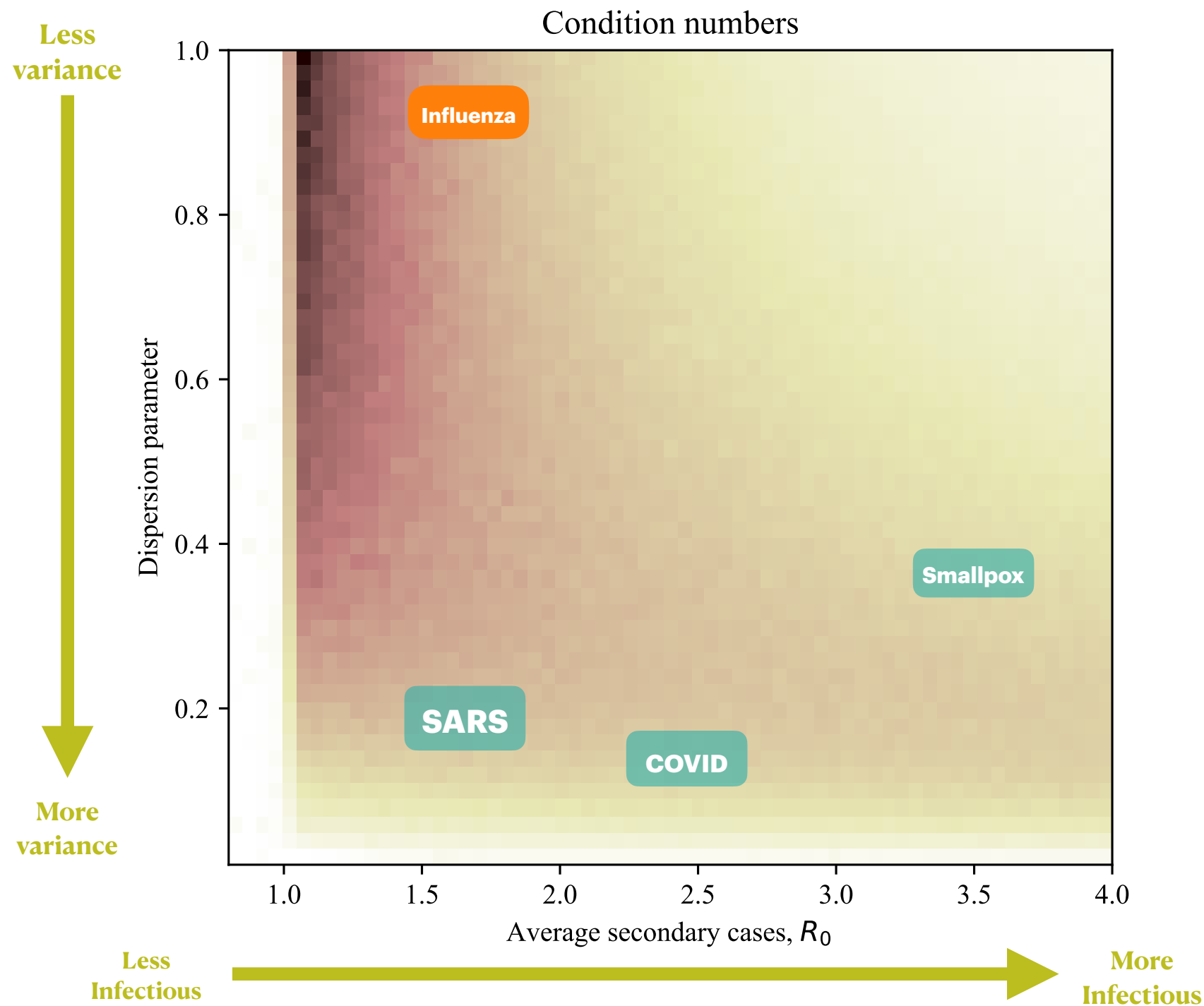
expected outbreak size



Our Method: We calculate the **sensitivity** of the **outbreak size** to perturbations in the **degree distribution**

The statistical condition estimate(SCE) is a measures of the sensitivity of the **outbreak size** to **perturbations in distribution**

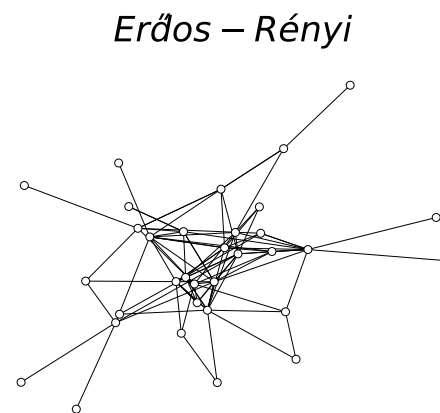
How do **dynamical parameters** of real world contagions effect **sensitivity**



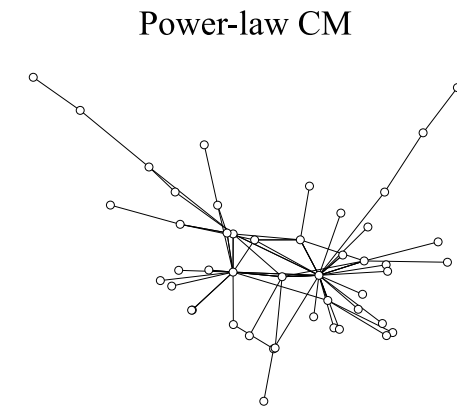
Contagions with **near critical R_0** and **high dispersion** like **influenza** are the most sensitive to measurement error

How does **graph structure** effect **sensitivity**?

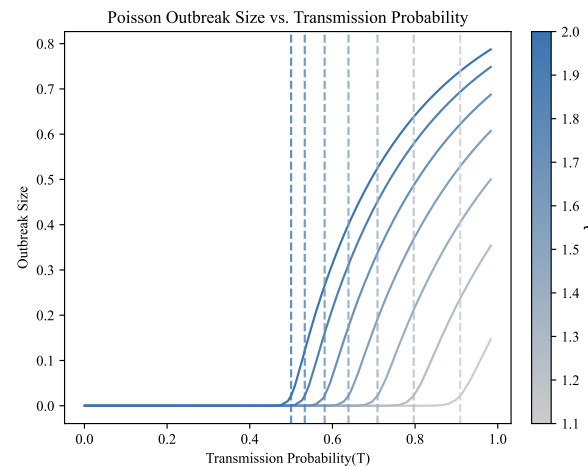
λ : the **density** of the graph



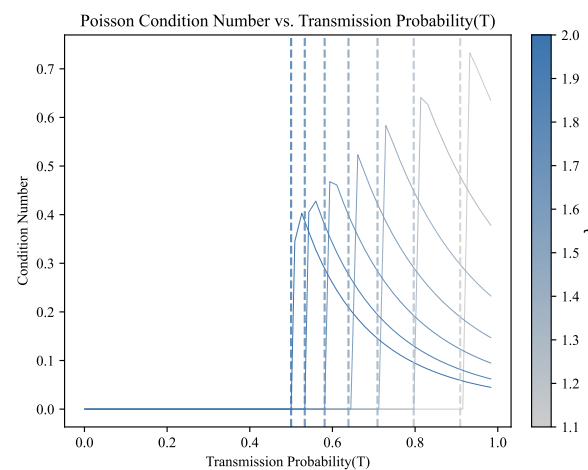
α : the **degree heterogeneity** of the graph



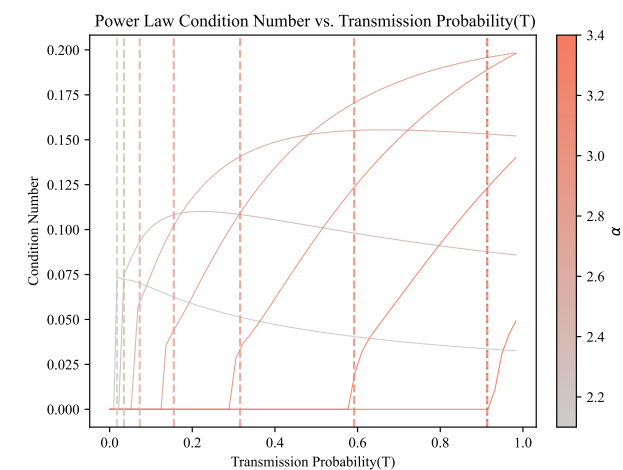
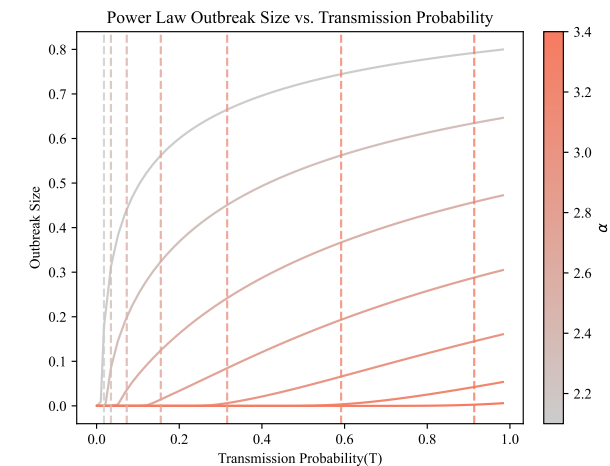
$\langle s \rangle$: Outbreak size



Statistical condition estimate



Sensitivity **jumps** at **critical transition** **regardless of density**.



For **highly heterogenous graphs** sensitivity **increases monotonically** with infection rate.

Jean Gabriel Young



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Mariah Boudreau



Chris M. Danforth



Laurent Hébert-Dufresne



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